EXPLICIT CODEWORDS FOR FOURTEEN OF THE TWENTY-TWO UNDETERMINED WEIGHTS OF THE REED–MULLER CODE $\mathrm{RM}(7,14)$

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Abstract. Leuenberger and Albrizzio (arXiv:2606.21425) show that the weight spectrum of the Reed–Muller code $\mathrm{RM}(7,14)$ contains every even integer between 312 and $2^{14}-312$, together with the classical small and large weights, with the possible exception of a set M of twenty-two weights, and write that they "were not able to determine the missing weights". We exhibit codewords for fourteen of the twenty-two: a Boolean function of degree 7 in 14 variables with six monomials and Hamming weight 354, functions with between 5 and 19 monomials of weights 8174, 8178, 8182, 8186, 8188, 8190, and their complements of weights 16030, 8210, 8206, 8202, 8198, 8196, 8194. Each codeword is given by its algebraic normal form, and its degree and its weight are checked from the definitions in the kernel of the Lean 4 proof assistant, without any library. We also record a mod4 law: a codeword of $\mathrm{RM}(7,14)$ has weight $\equiv 2 \pmod 4$ exactly when its algebraic normal form contains an odd number of complementary pairs of degree-7 monomials. The remaining four weights 322, 326, 330, 334 (and their complements) were not reached by four independent search families in about $3\cdot 10^9$ evaluations; we make no claim about their existence. The constructions and tables of the source are reproduced with no discrepancy.

1. Introduction

1.1. **Reed–Muller codes and their weight spectra.** A Boolean function in m variables is a map $f\colon \mathbb{F}_2^m \to \mathbb{F}_2$; it has a unique algebraic normal form $f = \sum_{L\subseteq\{1,\dots,m\}} a_L \prod_{l\in L} x_l$, and its algebraic degree is the largest $|L|$ with $a_L = 1$. The Reed–Muller code $\mathrm{RM}(r,m)$ is the set of truth tables of the Boolean functions in m variables of algebraic degree at most r. It is a binary linear code of length 2^m, dimension $\sum_{i\le r}\binom{m}{i}$ and minimum distance 2^{m-r}. The Hamming weight $\mathrm{wt}(f)$ of a codeword is the number of points of $\mathbb{F}_2^m$ at which f takes the value 1, and the *weight spectrum* $\mathcal{W}(\mathrm{RM}(r,m))$ is the set of Hamming weights that actually occur.

For the code studied here, $\mathrm{RM}(7,14)$, the length is 16384, the minimum distance is $d = 2^7 = 128$, and the dimension is $\sum_{i\le 7}\binom{14}{i} = 9908$. The spectrum is symmetric under $w \mapsto 2^{14} - w$, because $f \mapsto f+1$ is a bijection of the code that complements the truth table and does not raise the degree.

The classical results on the low end of the spectrum are due to Kasami and Tokura, whose theorem, in the words of their abstract, "completely characterizes the codewords of the νth-order Reed-Muller code whose weights are less than twice the minimum weight" [6], and to Kasami, Tokura and Azumi, who carried the classification up to $2.5d$: their abstract states that they "derive explicit formulas for the enumerators of all weights less than $2.5\,d$ of Reed–Muller codes, where d is the minimum weight" [5]. The second range contains the first. Carlet's abstract in [3] describes [5] the same way — it "precisely determines those codeword weights in Reed-Muller codes which lie between the minimum distance d and 2.5 times d" — and Lou and Wang [2] restate the boundary as "$B \subseteq [2^{c+1}, 2^{c+1} + 2^{c-1}]$ is given by Kasami, Tokura and Azumi", where indeed $2^{c+1} + 2^{c-1} = 2.5\cdot 2^c$. For $\mathrm{RM}(7,14)$ this threshold is $2.5d = 320$. We have read these abstracts, not the full texts; their provenance is recorded with the reference list.

Beyond the classical range, Carlet [3] determined the weight spectrum of $\mathrm{RM}(m-5,m)$ for $m \ge 10$, and Lou and Wang [2] that of $\mathrm{RM}(m-6,m)$ for $m \ge 12$:

$$\begin{aligned} A &= \{0, 64, 96, 112, 120, 124, 126, 128, 136, 144, 148\},\\ \mathcal{W}(\mathrm{RM}(m-6,m)) &= A \cup \{152 + 2i : 0 \le i \le 2^{m-1} - 152\} \cup \{2^m - a : a \in A\} \end{aligned} \tag{1}$$

Whether the same shape persists for every c is the open question behind both papers: is $\mathcal{W}(\mathrm{RM}(m-c,m))$, for $m \geq 2c$, of the form $\{0\}\cup A\cup B\cup C\cup\bar{B}\cup\bar{A}\cup\{2^m\}$, where $A \subseteq [2^c, 2^{c+1}]$ is the Kasami–Tokura set, $B \subseteq [2^{c+1}, 2^{c+1}+2^{c-1}]$ the Kasami–Tokura–Azumi set, C consists of *all* consecutive even integers between $2^{c+1}+2^{c-1}$ and $2^m-2^{c+1}-2^{c-1}$, and $\bar{A}, \bar{B}$ are the complements to 2^m? This is Conjecture 1 of [1], which labels it "Open question in" [3] and [2]. The question originates with Carlet and Solé: the concluding section of [4] states it as a conjecture, for $m > 2c-1$, and adds: "This conjecture is verified for $c = 1, 2, 3, 4$. The first open case is $c = 5$." Carlet [3] settled $c = 5$ and restated it as an open question — it "was later presented as an open question since it seemed to be very difficult to make a prediction by further study", as [2] puts it — and Lou and Wang, who settled $c = 6$, print it under a heading naming it the "Conjecture of" [4] and the "Open question of" [3]. One detail of the display above: the bullets printed in [4] carry 2^c where they are written here with 2^{c-1}; the form used here is the one printed in [3], [2] and [1], and it is the one that matches the $2.5d$ boundary of [5]. Carlet's abstract in [3] announces that it checks "a recent conjecture (in which we correct a misprint)". The code $\mathrm{RM}(7, 14)$ is the first member of the next family $c = 7$, at the smallest admissible $m = 2c$; Lou and Wang close their paper with "Further study may verify the conjecture for $RM(7, 14)$ and determine the weight spectrum of $RM(m-7, m)$."

1.2. **The source and its missing set.** Leuenberger and Albrizzio [1] extend the concatenation technique of Lou and Wang to four-term concatenations and prove the following. (Statement numbers attributed to [1] throughout are those printed in the arXiv PDF of version 1, the only version as of 7 September 2026.)

Proposition 1.1 ([1, Proposition 4])**.** *Let $\mathcal{W}_f$ be the weight spectrum of* $\mathrm{RM}(7, 14)$ *and let*

$$W_0 = \{0, 128, 192, 224, 240, 248, 252, 254, 256, 272, 288, 296, 304, 308\}.$$

Then

$$\big(W_0 \cup \{312 + 2i : 0 \leq i \leq 2^{13} - 312\} \cup \{2^{14} - w : w \in W_0\}\big) \setminus M \subseteq \mathcal{W}_f,$$

where the set of missing weights is

$$M = \{322,\ 326,\ 330,\ 334,\ 354,\ 8174,\ 8178,\ 8182,\ 8186,\ 8188,\ 8190,\ 8194,\ 8196,\ 8198,\ 8202,\ 8206,\ 8210,\ 16030,\ 16050,\ 16054,\ 16058,\ 16062\}.$$

(The displayed statement in the PDF writes W for W_0 in the complement term; the recapitulation at the end of its proof writes W_0.)

The set M is closed under $w \mapsto 2^{14} - w$, so it consists of eleven complementary pairs, with representatives $322, 326, 330, 334, 354$ below 2^{13} and $8174, 8178, 8182, 8186, 8188, 8190$ just below the midpoint $2^{13} = 8192$. Every element of M lies strictly above $2.5d = 320$ and strictly below $2^{14} - 320$; none of them is touched by the classical classification. Concerning M the authors write, in their Conclusion:

> "We were not able to determine the missing weights by using the concatenation technique used in this paper."

and, in a closing Remark:

> "Because of the way the weights are constructed, the list of missing weights M could be achieved by finding the following 8 weights: $\{322, 326, 330, 334, 354, 4378, 4380, 4382\}$."

The three mid-range weights of the Remark are not themselves in M: $4378 = 314 + 4 \cdot 1016$ and $4382 = 4254 + 128$ lie in the source's set $S_{2 \bmod 4}$ and $4380 = 304 + 4 \cdot 1019$ in its $S_{0 \bmod 4}$, so Proposition 1.1 already places them in the spectrum. Our reading of the Remark is that a codeword of one of these weights *inside the concatenation frame* could be combined with the source's "$+C$" step (adding a function g_0 of weight $c \in C = \{128, 192, 224, 240, 248, 256, 272\} \cup \{288 + 8i : 0 \leq i \leq 440\}$ on the first block) to reach the mid-range members of M: $8174 = 4382 + 3792$, $8178 = 4378 + 3800$, $8182 = 4382 + 3800$, $8186 = 4378 + 3808$, $8188 = 4380 + 3808$, $8190 = 4382 + 3808$, with $3792, 3800, 3808 \in C$. We do not use that route.

Each element of M is an existence question: is there a Boolean function of degree at most 7 in 14 variables whose truth table has that weight? An affirmative answer for all of M would confirm the shape asked for in Conjecture 1 of [1] for $c = 7$, $m = 14$.

1.3. **Results.** We answer fourteen of the twenty-two questions affirmatively, by exhibiting the codewords.

(a) There is a Boolean function of degree 7 in 14 variables with exactly six monomials whose weight is 354 (Theorem 3.1).
(b) There are Boolean functions of degree ≤ 7 in 14 variables, with 14, 15, 12, 14, 5 and 19 monomials respectively, of weights 8174, 8178, 8182, 8186, 8188 and 8190 (Theorem 3.2).
(c) Complementing them gives weights 16030, 8210, 8206, 8202, 8198, 8196, 8194 (Corollary 3.3).

Hence $M \cap \mathcal{W}(\mathrm{RM}(7, 14))$ has at least fourteen elements, and the set of weights left undetermined by Proposition 1.1 shrinks from twenty-two to eight:

$$\{322, 326, 330, 334\} \cup \{16062, 16058, 16054, 16050\}.$$

Equivalently, of the eight weights in the source's Remark, four remain: $322, 326, 330, 334$.

The certificates are the algebraic normal forms themselves, listed in Appendix A. For each one the degree bound and the weight are established by a computation from the definitions, carried out inside the kernel of the Lean 4 proof assistant (Section 7); nothing is imported from a mathematical library, and the model is the literal one: a codeword is a list of monomials, the value at a point is the sum of the monomials evaluated there, and the weight is the count of points at which the value is 1.

Two further items accompany the main results. First, a mod4 law (Lemma 4.1, Proposition 4.2): the weight of a codeword of $\mathrm{RM}(7, 14)$ is congruent modulo 4 to twice the number of complementary pairs $\{u, \bar{u}\}$ of degree-7 monomials in its algebraic normal form. All five small missing weights are $\equiv 2 \pmod 4$, so any codeword of one of these weights carries an odd number of such pairs. Second, a recomputation of everything the source establishes by hand (Section 5): the weight formulas of its Lemmas 2, 4 and 7, all eighteen rows of its Tables 1 and 2, its Proposition 2, and the codewords of its Lemmas 3, 5 and 8, all reproduced with no discrepancy, nine of them carried into the formal development as explicit codewords.

What is *not* claimed is as important. The four weights 322, 326, 330, 334 and their complements were searched for by four independent methods at a cost of about 1.4 processor-hours and roughly $3 \cdot 10^9$ evaluations, and were not found, while every neighbouring weight the same searchers were asked for was found within seconds. The search space has 2^{9908} elements; no exhaustive method is available, and the one exhaustive statement we can make (no codeword with at most four monomials has any of the five small weights) is demonstrably too weak to distinguish absence from difficulty, since 354 fails it too and exists. Section 6 reports the searches as a measurement. We make no claim, and intend no suggestion, that any of the four weights is absent from $\mathcal{W}(\mathrm{RM}(7, 14))$.

1.4. **Organisation.** Section 2 fixes notation and the bit-mask model in which the certificates are stated. Section 3 states the main results. Section 4 proves the mod4 law. Section 5 recomputes the source's own constructions and records the negative controls of the certificate checker. Section 6 reports the searches for the four remaining weights, and Section 7 describes the formal verification. Appendix A lists every certificate.

2. Preliminaries

2.1. **Boolean functions and the bit-mask model.** We write B_m for the set of Boolean functions in m variables. A monomial is $x_L = \prod_{l \in L} x_l$ for $L \subseteq \{1, \ldots, m\}$, with $x_\emptyset = 1$; every $f \in B_m$ is uniquely a sum (over $\mathbb{F}_2$) of distinct monomials, its algebraic normal form (ANF), and $\deg f$ is the largest $|L|$ occurring. The truth table of f is the vector $(f(x))_{x \in \mathbb{F}_2^m}$, and $\mathrm{wt}(f) = |\operatorname{supp} f|$ with $\operatorname{supp} f = \{x : f(x) = 1\}$. The Reed–Muller code $\mathrm{RM}(r, m) = \{f \in B_m : \deg f \leq r\}$ is a linear code of length 2^m. We use the following standard facts.

- $\mathrm{wt}(f + 1) = 2^m - \mathrm{wt}(f)$, and $\deg(f + 1) = \deg f$ for $f \neq 0, 1$; so $\mathcal{W}(\mathrm{RM}(r, m))$ is symmetric about 2^{m-1}.

- $\mathrm{wt}(f+g) = \mathrm{wt}(f) + \mathrm{wt}(g) - 2\,|\operatorname{supp} f \cap \operatorname{supp} g|$.
- Degree and weight are invariant under affine changes of variables $x \mapsto Ax+b$ with $A \in \mathrm{GL}_m(\mathbb{F}_2)$, $b \in \mathbb{F}_2^m$.

We now describe the model in which every statement of this paper is formalised. A point of $\mathbb{F}_2^{14}$ is identified with an integer x with $0 \le x < 2^{14}$, bit i of x (for $0 \le i < 14$) being the value of the variable x_{i+1}. A monomial x_L is identified with its *mask* $u = \sum_{l \in L} 2^{l-1}$, an integer $0 \le u < 2^{14}$ whose binary digits mark the variables of the monomial; $|u|$ denotes the number of 1-bits of u, which is the degree of the monomial, and $\bar{u} = (2^{14}-1) - u$ is the mask of the complementary set of variables. Writing $u \wedge x$ for the bitwise AND,

$$x_L(x) = 1 \iff u \wedge x = u.$$

Definition 2.1. Let A be a finite list of masks (a list of integers in $[0, 2^{14})$).

- $F_A \colon \mathbb{F}_2^{14} \to \mathbb{F}_2$ is the function $F_A(x) = \bigoplus_{u \in A} [\, u \wedge x = u \,]$; a mask repeated an even number of times cancels, exactly as in the polynomial.
- $\mathrm{wt}(A) = \#\{x : 0 \le x < 2^{14},\ F_A(x) = 1\}$ is the Hamming weight of F_A.
- *A presents a codeword of* $\mathrm{RM}(r, 14)$, written $A \in \mathrm{RM}(r, 14)$, when every $u \in A$ satisfies $u < 2^{14}$ and $|u| \le r$.
- $\overline{A}$ (the complement) is A with the mask 0 removed if present, and with 0 prepended otherwise; thus $F_{\overline{A}} = F_A + 1$.
- $\mathrm{cp}(A)$ is the number of unordered pairs $\{u, \bar{u}\}$ with $u, \bar{u} \in A$ and $|u| = |\bar{u}| = 7$.

When A has no repeated mask, F_A is the Boolean function whose ANF is $\sum_{u \in A} x_{L(u)}$, and $A \in \mathrm{RM}(r, 14)$ says exactly that $\deg F_A \le r$. Every certificate in this paper is such a duplicate-free list. In the formal development the five items of Definition 2.1 are the functions `evalAt`, `wt`, `isRM`, `compl` and `compPairs` on `List Nat`, defined by the same words; see Section 7.

2.2. The weight of a sum of monomials.

Lemma 2.2 (inclusion–exclusion)**.** *Let A be a set of k distinct masks in $[0, 2^m)$. Then*

$$\mathrm{wt}(F_A) \;=\; \sum_{\emptyset \ne T \subseteq A} (-2)^{|T|-1}\, 2^{\,m - |\bigcup T|},$$

where $\bigcup T$ is the bitwise OR of the masks in T and $|\bigcup T|$ its number of 1-bits.

Proof. $\operatorname{supp} F_A$ is the symmetric difference of the supports $S_u = \{x : u \wedge x = u\}$, $u \in A$, and for any finite family of sets $|S_1 \triangle \cdots \triangle S_k| = \sum_{\emptyset \ne T} (-2)^{|T|-1} |\bigcap_{j \in T} S_j|$ (induct on k using $|S \triangle S'| = |S| + |S'| - 2|S \cap S'|$). Finally $\bigcap_{u \in T} S_u = \{x : x \supseteq \bigcup T\}$ has $2^{m - |\bigcup T|}$ elements. □

For k monomials the formula has $2^k - 1$ terms and depends only on the sizes of the 2^k Venn regions of the variable sets; it is the engine of the searches of Section 6. It is used in the proof of Lemma 4.1; it is not used by the formal verification, which counts points directly.

2.3. The concatenation frame of the source.

For $f_1, f_2 \in B_m$ the source writes $(f_1 \| f_2) = (x_{m+1} + 1) f_1 + x_{m+1} f_2 \in B_{m+1}$, and similarly for four functions with two new variables. Its Lemma 1 states that $g = g_0 \| g_1 \| g_2 \| (g_1 + g_2 + g_3)$ lies in $\mathrm{RM}(7, 14)$ whenever $g_0, g_3 \in \mathrm{RM}(5, 12)$ and $g_1, g_2 \in \mathrm{RM}(6, 12)$, and its proof gives the expansion

$$g = (1 + x_{13})(1 + x_{14})\, g_0 + x_{13}\, g_1 + x_{14}\, g_2 + x_{13} x_{14}\, g_3. \tag{2}$$

Thus for $g_0 = g_3 = 0$ the ANF of g is $x_{13} \cdot \mathrm{ANF}(g_1) + x_{14} \cdot \mathrm{ANF}(g_2)$; the variants $0 \| (g_1 + 1) \| g_2 \| (g_1 + g_2)$ (Table 1 of the source) and $0 \| g_1 \| g_2 \| (g_1 + g_2 + 1)$ (its Table 2 and Proposition 2) pick up the extra terms $x_{13} + x_{13}x_{14}$ and $x_{13}x_{14}$ respectively. This is how the codewords of Section 5 were written down from the source's tables. In this frame

$$\mathrm{wt}\big(0 \| g_1 \| g_2 \| (g_1 + g_2)\big) = \mathrm{wt}(g_1) + \mathrm{wt}(g_2) + \mathrm{wt}(g_1 + g_2) = 2\,|\operatorname{supp} g_1 \cup \operatorname{supp} g_2|. \tag{3}$$

3. Main results

Throughout, c_w denotes the list of masks given in Appendix A under that name; each is a duplicate-free list, so F_{c_w} is a Boolean function in 14 variables whose ANF has exactly the listed monomials.

Theorem 3.1. *The list* $c_{354} = [7245, 7493, 9130, 9138, 10009, 14097]$ *presents a codeword of* $\mathrm{RM}(7,14)$ *(each of its six masks has exactly seven* 1*-bits), and* $\mathrm{wt}(c_{354}) = 354$*. Consequently* $354 \in \mathcal{W}(\mathrm{RM}(7,14))$*.*

In the variables, $F_{c_{354}}$ is the degree-7 polynomial

$$F_{c_{354}} = x_1x_3x_4x_7x_{11}x_{12}x_{13} + x_1x_3x_7x_9x_{11}x_{12}x_{13} + x_2x_4x_6x_8x_9x_{10}x_{14} + x_2x_5x_6x_8x_9x_{10}x_{14} + x_1x_4x_5x_9x_{10}x_{11}x_{14} + x_1x_5x_9x_{10}x_{11}x_{13}x_{14}.$$

Theorem 3.2. *The lists* c_{8174}*,* c_{8178}*,* c_{8182}*,* c_{8186}*,* c_{8188}*,* c_{8190} *of Appendix A, with* 14*,* 15*,* 12*,* 14*,* 5 *and* 19 *masks respectively, present codewords of* $\mathrm{RM}(7,14)$*, and*

$$\mathrm{wt}(c_{8174}) = 8174, \quad \mathrm{wt}(c_{8178}) = 8178, \quad \mathrm{wt}(c_{8182}) = 8182,$$
$$\mathrm{wt}(c_{8186}) = 8186, \quad \mathrm{wt}(c_{8188}) = 8188, \quad \mathrm{wt}(c_{8190}) = 8190.$$

Consequently $\{8174, 8178, 8182, 8186, 8188, 8190\} \subseteq \mathcal{W}(\mathrm{RM}(7,14))$*.*

The shortest of these, of weight $8188 = 2^{13} - 4$, is

$$F_{c_{8188}} = x_6 + x_3x_4x_9x_{11}x_{12} + x_1x_2x_6x_7x_8x_{11}x_{13} + x_1x_2x_{11}x_{14} + x_3x_4x_5x_{10}x_{11}x_{14}.$$

Corollary 3.3. *The complements* $\overline{c_{354}}$*,* $\overline{c_{8174}}$*,* $\overline{c_{8178}}$*,* $\overline{c_{8182}}$*,* $\overline{c_{8186}}$*,* $\overline{c_{8188}}$*,* $\overline{c_{8190}}$ *present codewords of* $\mathrm{RM}(7,14)$ *of weights* 16030*,* 8210*,* 8206*,* 8202*,* 8198*,* 8196*,* 8194 *respectively. Consequently these seven weights lie in* $\mathcal{W}(\mathrm{RM}(7,14))$*.*

Proof of Theorems 3.1, 3.2 and Corollary 3.3. Each statement is a finite computation on an explicit list, and each was carried out twice: once by an independent Python re-implementation from the truth table, and once inside the Lean 4 kernel from Definition 2.1. The degree assertion $A \in \mathrm{RM}(7,14)$ is a check of at most 19 bit counts and is decided by the kernel with no axiom at all (`decide`). The weight assertion evaluates F_A at each of the 2^{14} points and counts the ones; this count is evaluated by the compiled code of the Lean runtime (`native_decide`). Corollary 3.3 is the same computation on the complemented lists (0 prepended: none of the seven certificates contains the empty monomial), so it is certified, not derived; the derivation $\mathrm{wt}(f+1) = 2^{14} - \mathrm{wt}(f)$ is the reason the numbers come out as they do. Section 7 lists the declarations and their axioms. □

Corollary 3.4. $|M \cap \mathcal{W}(\mathrm{RM}(7,14))| \geq 14$*, and every element of* $W_0 \cup \{312 + 2i : 0 \leq i \leq 2^{13} - 312\} \cup \{2^{14} - w : w \in W_0\}$*, with the possible exception of the eight weights* 322*,* 326*,* 330*,* 334*,* 16050*,* 16054*,* 16058*,* 16062*, belongs to* $\mathcal{W}(\mathrm{RM}(7,14))$*.*

Remark 3.5 (How the certificates were found)*.* Nothing about the proofs depends on this, but the reader may want to know where the lists came from. The mid-range certificates of Theorem 3.2 (and those of Proposition 5.3 below) were found by a plain steepest descent in ANF space: the truth table is kept as 256 machine words, all 9908 monomials of degree ≤ 7 are precomputed, a move toggles one monomial, and the change of weight is $2^{14-|u|} - 2\,|\operatorname{supp} f \cap \operatorname{supp} x_u|$; each target was reached within seconds. This searcher is not strong at the low end of the spectrum (as a calibration, it failed to find the known weight 162 of $\mathrm{RM}(6,12)$). The weight-354 certificate was found by a randomised local search over sums of indicator functions of k affine subspaces of dimension 7 of $\mathbb{F}_2^{14}$ — each such indicator is a product of seven affine forms, hence a codeword of $\mathrm{RM}(7,14)$, and these indicators generate the code — followed by an affine change of variables chosen by hill-climbing to minimise the number of ANF monomials. Weight and degree are affine invariants, so the sparse representative is a codeword of the same weight. The codeword found had 3874 monomials; the sparsification reduced it to six in 90 s. Once six monomials were known to suffice, a local search over six-monomial ANFs driven by Lemma 2.2 found a six-monomial witness independently in under a second. We stress that the first version of the subspace searcher had a defective rank test, accepted degenerate "subspaces" of lower dimension (whose indicators have degree > 7), and reported all five small missing weights as found; the

independent recomputation from the truth table gave weight 224 for its "322" and exposed the defect. Nothing in this paper is believed until it has been recomputed from the definitions by an independent implementation, and the Lean check is the last of these.

4. A mod4 law

All five small weights of M are $\equiv 2 \pmod 4$, as are $8174, 8178, 8182, 8186, 8190$; only 8188 (and its complement 8196) is $\equiv 0$. The following elementary observation says which codewords can have weight $\equiv 2 \pmod 4$.

Lemma 4.1. *Let A be a set of distinct masks in $[0, 2^{14})$, each with $|u| \le 7$. Then*

$$\mathrm{wt}(F_A) \;\equiv\; 2 \cdot \mathrm{cp}(A) \pmod 4,$$

where $\mathrm{cp}(A)$ is the number of unordered pairs $\{u, \bar{u}\} \subseteq A$ with $|u| = 7$. In particular $\mathrm{wt}(F_A) \equiv 2 \pmod 4$ if and only if A contains an odd number of complementary pairs of degree-7 monomials.

Proof. Reduce the expansion of Lemma 2.2 modulo 4. A singleton term is $2^{14-|u|}$ with $|u| \le 7$, hence divisible by 2^7. A term with $|T| \ge 3$ carries the factor $2^{|T|-1}$, divisible by 4. A term with $T = \{u, v\}$ is $-2 \cdot 2^{14-|u \cup v|}$, which is $\equiv 2 \pmod 4$ exactly when $|u \cup v| = 14$; since $|u \cup v| \le |u| + |v| \le 14$, this forces $|u| = |v| = 7$ and $u \cap v = \emptyset$, that is $v = \bar{u}$. Each complementary pair contributes $-2 \equiv 2$, and every other term contributes 0. □

The lemma is certainly folklore; we record it because it constrains the four open weights. Its instance on every certificate of this paper is part of the formal development:

Proposition 4.2. *For each of the ten lists A among c_{354}, c_{4378}, c_{4380}, c_{4382}, c_{8174}, c_{8178}, c_{8182}, c_{8186}, c_{8188}, c_{8190} of Appendix A,*

$$\mathrm{wt}(A) \bmod 4 \;=\; 2 \cdot (\mathrm{cp}(A) \bmod 2).$$

Proof. A direct evaluation of both sides for the ten lists (`native_decide`). The values are $\mathrm{cp} = 1$ for c_{354}, c_{4378}, c_{4382}, c_{8174}, c_{8178}, c_{8182}, c_{8186}, c_{8190} and $\mathrm{cp} = 0$ for c_{4380}, c_{8188}, matching the residues $2, 2, 0, 2, 2, 2, 2, 2, 0, 2$ of the weights. Outside the formal development the law was also tested against 400 random codewords of $\mathrm{RM}(7, 14)$ in Python, with no mismatch. □

Corollary 4.3. *A codeword of $\mathrm{RM}(7, 14)$ of weight 322, 326, 330 or 334, if one exists, has an odd number of complementary pairs $\{x_L, x_{\bar{L}}\}$ of degree-7 monomials in its algebraic normal form. The cheapest carrier of one such pair, $x_1 \cdots x_7 + x_8 \cdots x_{14}$, has weight 254 (the value of the source's Lemma 5, and the third control of Proposition 5.4).*

5. The source's constructions, recomputed

Before searching for new codewords we recomputed everything the source establishes by hand, as a check on our reading of its frame (2). All of it agrees.

A labelled computation, outside the formal development. A C program enumerated, over $\mathbb{F}_2^{12}$ truth tables, all pairs of degree-6 monomials (the source's Lemma 2, $\mathrm{wt} = 128 - 2^{c+1}$ with $c = |I \cap J|$; the printed weight set $\{0, 64, 96, 112, 120, 124, 126\}$), all 924^2 pairs (J, K) with $I = \{1, \ldots, 6\}$ for its three-monomial formula (Lemma 4; the printed 19-element weight set), and $3 \cdot 10^6$ random triples (J, K, L) for its four-monomial formula (Lemma 7; the sampled weight set equals the printed 41-element set). A second program rebuilt all eight rows of Table 1 and all ten rows of Table 2 column by column (including the rows 4358 and 4362), and both cases of Proposition 2. There were no mismatches in any of the formulas, weight sets, table columns or totals. One typographical point: Lemma 5 of the source prints the monomial $x_1x_7x_8x_9x_{10}x_{11}$ twice in g_1, so the two copies cancel; its stated $\mathrm{wt}(g_1) = 64$ is the weight of the surviving single monomial $x_1 \cdots x_6$, and the arithmetic $64 + 126 + 64 = 254$ is right.

Codewords carried into the formal development. Nine of the source's codewords were written down as ANFs through (2) and certified.

Proposition 5.1. *The lists c_{254}, c_{314}, c_{342}, c_{350}, c_{362} of Appendix A present codewords of* $\mathrm{RM}(7,14)$ *of weights* 254, 314, 342, 350, 362*: the five weights the source establishes by hand in its Lemmas 5, 3 and 8, rebuilt from the functions g_1, g_2 printed there as $x_{13}\,g_1 + x_{14}\,g_2$.*

Proposition 5.2. *The lists c_{4238}, c_{4242}, c_{4372}, c_{4376} of Appendix A present codewords of* $\mathrm{RM}(7,14)$ *of weights* 4238, 4242, 4372, 4376*: the first row of the source's Table 2, the first row of its Table 1, and the two cases of its Proposition 2.*

The three mid-range weights of the source's closing Remark also received explicit codewords. These weights are already inside the set of Proposition 1.1, as computed in Section 1.2; what is new is the witness, not the fact.

Proposition 5.3. *The lists c_{4378}, c_{4380}, c_{4382} of Appendix A, with* 16, 14 *and* 14 *masks, present codewords of* $\mathrm{RM}(7,14)$ *of weights* 4378, 4380, 4382.

Negative controls. A checker that accepts everything proves nothing, so the formal development also records that the two halves of each certificate check can fail.

Proposition 5.4. *In the model of Definition 2.1:*

(1) *The lists* [16383] *(the monomial $x_1 \cdots x_{14}$) and* [255] *(the monomial $x_1 \cdots x_8$) do not present codewords of* $\mathrm{RM}(7,14)$.
(2) *The list* $[7245, 7493, 9130, 9139, 10009, 14097]$ *has weight* 354 *but does not present a codeword of* $\mathrm{RM}(7,14)$*: the mask* 9139 *has eight* 1*-bits. (It differs from c_{354} in one bit of one mask.)*
(3) $\mathrm{wt}([\,]) = 0$*;* $\mathrm{wt}([127]) = 128$*, the minimum distance; and* $\mathrm{wt}([127, 16256]) = 254$*, the weight of $x_1 \cdots x_7 + x_8 \cdots x_{14}$.*
(4) $\mathrm{wt}([7245, 7493, 9130, 9138, 10009]) = 382$ *and* $\mathrm{wt}([3340, 5347, 9219, 9756]) = 1476$*: deleting the last monomial of c_{354}, or the first monomial of c_{8188}, changes the weight.*

Item (1) is decided by the kernel with no axiom. Item (2) refutes a too-large claim (right weight, wrong degree), item (4) refutes a too-small claim (a corrupted certificate does not pass), and item (3) shows the counter is honest at the bottom of the spectrum.

6. The four unreached weights

Everything in this section is a report on computations outside the formal development; none of it is a theorem of this paper, and nothing here is a claim of non-existence.

6.1. **The searches.** Four independent search families were run for the weights $322, 326, 330, 334$ (searching for a weight w or for $2^{14} - w$ is the same problem). All programs are Mathlib-free C; the figures are as recorded during the run.

(i) *Exhaustive enumeration for at most four monomials.* By Lemma 2.2 the weight of a k-monomial ANF depends only on the sizes of the 2^k Venn regions of the k variable sets, so enumerating all region-size vectors with each set of size ≤ 7 is an exhaustive search over k-monomial codewords up to permutation of the variables. For $k = 3$ there are 20,708 cases, for $k = 4$ there are 8,568,777 (in 3.6 s). Outcome: none of $322, 326, 330, 334, 354$ is the weight of a codeword with at most four monomials, whereas 312, 314, 316, 318, 320, 324, 328, 332, 336, 338, 340, 342, 344, 348, 350, 352, 356 all are.
(ii) *Steepest descent in ANF space*, as in Remark 3.5, with 10^5 restarts. Best weight reached: within 2 of each target.
(iii) *Local search over sums of k indicators of 7-dimensional affine subspaces, $k = 4, \ldots, 8$,* moves reseating one subspace's offset, one basis vector or the whole subspace, with a Gaussian-elimination rank test; up to twelve runs of 120 s per weight, about $2.4 \cdot 10^9$ accepted or rejected moves, about 1.2 processor-hours. Best weight reached: within 2. This is the family that found 354.

(iv) *Local search over k-monomial ANFs*, $k = 5, \dots, 9$, each evaluation through the 2^k-term formula of Lemma 2.2 at about $2 \cdot 10^6$ evaluations per second; about $6 \cdot 10^8$ evaluations, about 0.15 processor-hours; run plain and in a variant that fixes the complementary pair $x_1 \cdots x_7$, $x_8 \cdots x_{14}$ so that, by Lemma 4.1, every candidate has weight $\equiv 2 \pmod 4$. Best weight reached: within 2.

The total cost of everything reported in this paper was about 1.6 processor-hours, of which about 1.4 were these searches; no process exceeded 30 MB of memory.

6.2. **Calibration.** Family (iv) with $k = 6$ finds the neighbouring weights 320, 324, 346, 354, 358, 362 and 370 within seconds each, and never reaches 322, 326, 330 or 334. On the other side, the exhaustive statement (i) is not evidence of absence: 354 fails it and is a weight of the code (Theorem 3.1), by a six-monomial codeword. What is on record is therefore a measurement, not an argument: a searcher that finds every neighbouring weight it is asked for in seconds does not find these four in about $3 \cdot 10^9$ evaluations. The search space is 2^{9908}; no exhaustive method over it is within reach.

6.3. **Two partial obstructions, for 322 only.** The following two hand arguments were recorded during the search; they are not formalised and they concern only specific constructions, not the code.

Remark 6.1. No sum of three indicator functions of 7-dimensional affine subspaces of $\mathbb{F}_2^{14}$ has weight 322. Indeed, for affine subspaces A, B, C of size 128,

$$\mathrm{wt}(1_A + 1_B + 1_C) = 384 - 2X + 4Y, \qquad X = |A \cap B| + |A \cap C| + |B \cap C|, \quad Y = |A \cap B \cap C|,$$

so weight 322 needs $X - 2Y = 31$. Each pairwise intersection is empty or an affine subspace, so its size is 0 or a power of 2, and X odd forces one or three of them to have size 1. Three gives $X = 3$, and $X - 2Y = 31$ is then impossible since $Y \geq 0$. One, say $|A \cap B| = 1$, gives $Y \leq 1$; $Y = 0$ needs $|A \cap C| + |B \cap C| = 30$, not a sum of two powers of two or zero; $Y = 1$ needs $|A \cap C| + |B \cap C| = 32$ with both nonempty, hence $16 + 16$, so the direction spaces of $A \cap C$ and $B \cap C$ are 4-dimensional subspaces of the 7-dimensional direction space of C meeting trivially — impossible.

Remark 6.2. No codeword of the form $0\|g_1\|g_2\|(g_1 + g_2)$ with $g_1, g_2 \in \mathrm{RM}(6, 12)$ — the frame of the source's Constructions 1, 2 and 3, which is where its set $S_{2 \bmod 4,1} = \{254\} \cup \{314 + 4i : 0 \leq i \leq 58\} \setminus \{322, 326, 330, 334, 354\}$ of small weights $\equiv 2 \pmod 4$ comes from — has weight 322. By (3) such a weight is $\mathrm{wt}(g_1) + \mathrm{wt}(g_2) + \mathrm{wt}(g_1 + g_2)$, and the three summands satisfy the triangle inequality, so each is at most 161. By Lou and Wang's theorem (1) at $m = 12$ (Lemma 11 of the source),

$$\mathcal{W}(\mathrm{RM}(6, 12)) \cap [0, 161] = \{0, 64, 96, 112, 120, 124, 126, 128, 136, 144, 148, 152, 154, 156, 158, 160\},$$

and a check over this 16-element set finds no three elements (repetition allowed) summing to 322. The same test does not exclude the other three: $326 = 64 + 126 + 136$, $330 = 64 + 112 + 154$, and 334 admits $64 + 112 + 158$, $64 + 126 + 144$ and $96 + 112 + 126$; nor does it exclude $354 = 64 + 136 + 154$, which exists. So this remark explains why the source's frame misses 322, and says nothing about why it misses the other four.

6.4. **What remains.** The weights $322, 326, 330, 334$ and their complements $16062, 16058, 16054, 16050$ are, to our knowledge, the only *even* integers in $[312, 2^{14} - 312]$ not yet known to lie in $\mathcal{W}(\mathrm{RM}(7, 14))$ (no odd integer does: $\mathrm{RM}(7, 14)$ is contained in the even weight code $\mathrm{RM}(13, 14)$). Corollary 4.3 says a codeword of one of these weights must carry an odd number of complementary degree-7 pairs; forcing three such pairs, or searching the $(u\|u + v)$ frame $\mathrm{wt}(u\|u + v) = \mathrm{wt}(u) + \mathrm{wt}(u + v)$ with $u \in \mathrm{RM}(7, 13)$, $v \in \mathrm{RM}(6, 13)$ over cosets of $\mathrm{RM}(6, 13)$, are the directions not yet tried. We also note that $\mathcal{W}(\mathrm{RM}(6, 13)) \subseteq \mathcal{W}(\mathrm{RM}(7, 14))$ via $f \mapsto (1 + x_{14})f$, which preserves the weight and raises the degree by at most one; the spectrum of $\mathrm{RM}(6, 13)$ is the same $c = 7$ family one variable down (below the range $m \geq 2c$ of Conjecture 1) and is equally unknown to us. An impossibility proof for any of the four weights would refute the shape asked for in Conjecture 1 of [1] at $c = 7$; we have no evidence in either direction beyond the measurement above.

7. Verification

Theorems 3.1 and 3.2, Corollary 3.3 and Propositions 4.2, 5.1, 5.2, 5.3 and 5.4 have been verified in Lean 4 [7]. The development is three modules that import nothing from Mathlib (only the core `Lean` library, through a one-line tagging attribute of the project): `Core` (the model of Definition 2.1 as the functions `bits`, `evalAt`, `wtAux`, `wt`, `isRM`, `compl`, `compPairs` over `Nat` and `List Nat`, with the constant `npts = 16384`, and the four control theorems of Proposition 5.4), `Source` (the nine certificates of Propositions 5.1 and 5.2 and the two theorems `source_lemma_weights`, `source_table_weights`) and `Missing` (the ten certificates of Theorems 3.1, 3.2 and Proposition 5.3 and the five theorems `weight_354`, `weights_8174_to_8190`, `complementary_weights`, `weights_4378_4380_4382`, `mod4_law_on_witnesses`). In all there are 38 declarations: 8 definitions of the model, 19 certificate definitions (the lists of Appendix A, verbatim), and 11 theorems, each a conjunction of equalities of the forms "`isRM` 7 $c = b$" and "`wt` $c = w$" over explicit lists (Proposition 4.2 is a single Boolean "`all`" over its ten lists).

Every degree assertion is discharged by `decide`, that is, by reduction inside the kernel, and contributes no axiom. Every weight assertion is a count over 2^{14} points and is discharged by `native_decide`, whose trust is the compiled evaluation of the definitions above; this is the only use of compiled evaluation. A print of the axiom set of each theorem returns `propext` together with the `native_decide` axioms generated for that theorem's weight conjuncts, one per conjunct, and nothing else; the single exception is the degree-only control, which depends on no axiom at all.

theorem	weight conjuncts (`native_decide` axioms)
`isRM_rejects_high_degree` (Prop. 5.4(1))	0 (no axioms)
`near_miss_degree_eight` (Prop. 5.4(2))	1
`wt_small_cases` (Prop. 5.4(3))	3
`perturbed_certificates` (Prop. 5.4(4))	2
`source_lemma_weights` (Prop. 5.1)	5
`source_table_weights` (Prop. 5.2)	4
`weight_354` (Thm. 3.1)	1
`weights_8174_to_8190` (Thm. 3.2)	6
`complementary_weights` (Cor. 3.3)	7
`weights_4378_4380_4382` (Prop. 5.3)	3
`mod4_law_on_witnesses` (Prop. 4.2)	1

There is no `sorry`, and neither `Classical.choice` nor `Quot.sound` appears. Checking the three modules one at a time took 2.4 s, 2.5 s and 4.5 s respectively.

Outside the formal development, and labelled as such where they occur, are the C and Python programs of Sections 5 and 6: the brute-force reproduction of the source's formulas and tables, the four search families with the costs stated there, the sparsification of Remark 3.5, the 400-codeword test of Lemma 4.1, and an independent Python re-check of the weight and degree of every certificate from its truth table (the check that caught the false positive of Remark 3.5). The source of the development and of the programs is currently held in a private repository: repository reference to be added at submission.

On the reference list. Statement, table and construction numbers attributed to [1] are those printed in the arXiv PDF of version 1 (14 pages, dated June 23, 2026 on its title page; submitted 19 June 2026); the arXiv abstract page titles the paper "On the Weight Spectrum of the Reed-Muller Codes $RM(7, 14)$" while the PDF omits the article. The two quotations in Section 1.2 are verbatim from that PDF. Lou and Wang's theorem (1) was read from their arXiv e-print (as content; no statement number of [2] is cited here) and, at $m = 12$, from Lemma 11 of [1]; the sentences quoted from [2] and [4] are verbatim from their arXiv e-prints, whose versions are pinned in the reference list. Every volume, issue, page range and DOI below was checked against Crossref, dblp and OpenAlex, which agree on all of them; note that the issue number of [2] is 11, not the 12 printed in the reference list of [1]. The sentence quoted from the abstract of [5] was read from an archived copy of the Elsevier article page (the live page is paywalled), and the sentences quoted from the abstracts of [6] and [3] from the IEEE Xplore records; the full texts of these three papers were not consulted.

Appendix A. The certificates

Each certificate is a list of masks; the mask u stands for the monomial $\prod_{i:\text{bit } i \text{ of } u \text{ is } 1} x_{i+1}$, and the codeword is the sum of the listed monomials over $\mathbb{F}_2$. The lists are exactly those in the formal development, in the same order. The first ten are the new witnesses of Theorems 3.1, 3.2 and Proposition 5.3; the last nine are the source's own constructions of Propositions 5.1 and 5.2. Every mask has at most seven 1-bits.

c_{354}: (6 monomials; masks 7245, 7493, 9130, 9138, 10009, 14097)

$$c_{354} = x_1x_3x_4x_7x_{11}x_{12}x_{13} + x_1x_3x_7x_9x_{11}x_{12}x_{13} + x_2x_4x_6x_8x_9x_{10}x_{14} + x_2x_5x_6x_8x_9x_{10}x_{14} + x_1x_4x_5x_9x_{10}x_{11}x_{14} + x_1x_5x_9x_{10}x_{11}x_{13}x_{14}.$$

c_{8174}: (14 monomials; masks 12, 32, 129, 179, 528, 1123, 1945, 3178, 3340, 4363, 9219, 9756, 14438, 15434)

$$c_{8174} = x_3x_4 + x_6 + x_1x_8 + x_1x_2x_5x_6x_8 + x_5x_{10} + x_1x_2x_6x_7x_{11} + x_1x_4x_5x_8x_9x_{10}x_{11} + x_2x_4x_6x_7x_{11}x_{12} + x_3x_4x_9x_{11}x_{12} + x_1x_2x_4x_9x_{13} + x_1x_2x_{11}x_{14} + x_3x_4x_5x_{10}x_{11}x_{14} + x_2x_3x_6x_7x_{12}x_{13}x_{14} + x_2x_4x_7x_{11}x_{12}x_{13}x_{14}.$$

c_{8178}: (15 monomials; masks 12, 32, 129, 179, 528, 1123, 1945, 3178, 3340, 4363, 9219, 9756, 11523, 14438, 15434)

$$c_{8178} = x_3x_4 + x_6 + x_1x_8 + x_1x_2x_5x_6x_8 + x_5x_{10} + x_1x_2x_6x_7x_{11} + x_1x_4x_5x_8x_9x_{10}x_{11} + x_2x_4x_6x_7x_{11}x_{12} + x_3x_4x_9x_{11}x_{12} + x_1x_2x_4x_9x_{13} + x_1x_2x_{11}x_{14} + x_3x_4x_5x_{10}x_{11}x_{14} + x_1x_2x_9x_{11}x_{12}x_{14} + x_2x_3x_6x_7x_{12}x_{13}x_{14} + x_2x_4x_7x_{11}x_{12}x_{13}x_{14}.$$

c_{8182}: (12 monomials; masks 32, 144, 177, 949, 1251, 2064, 3178, 3340, 4363, 9219, 9756, 15434)

$$c_{8182} = x_6 + x_5x_8 + x_1x_5x_6x_8 + x_1x_3x_5x_6x_8x_9x_{10} + x_1x_2x_6x_7x_8x_{11} + x_5x_{12} + x_2x_4x_6x_7x_{11}x_{12} + x_3x_4x_9x_{11}x_{12} + x_1x_2x_4x_9x_{13} + x_1x_2x_{11}x_{14} + x_3x_4x_5x_{10}x_{11}x_{14} + x_2x_4x_7x_{11}x_{12}x_{13}x_{14}.$$

c_{8186}: (14 monomials; masks 32, 144, 177, 244, 949, 1251, 2064, 3178, 3340, 4363, 9219, 9756, 14438, 15434)

$$c_{8186} = x_6 + x_5x_8 + x_1x_5x_6x_8 + x_3x_5x_6x_7x_8 + x_1x_3x_5x_6x_8x_9x_{10} + x_1x_2x_6x_7x_8x_{11} + x_5x_{12} + x_2x_4x_6x_7x_{11}x_{12} + x_3x_4x_9x_{11}x_{12} + x_1x_2x_4x_9x_{13} + x_1x_2x_{11}x_{14} + x_3x_4x_5x_{10}x_{11}x_{14} + x_2x_3x_6x_7x_{12}x_{13}x_{14} + x_2x_4x_7x_{11}x_{12}x_{13}x_{14}.$$

c_{8188}: (5 monomials; masks 32, 3340, 5347, 9219, 9756)

$$c_{8188} = x_6 + x_3x_4x_9x_{11}x_{12} + x_1x_2x_6x_7x_8x_{11}x_{13} + x_1x_2x_{11}x_{14} + x_3x_4x_5x_{10}x_{11}x_{14}.$$

c_{8190}: (19 monomials; masks 16, 23, 31, 32, 145, 2094, 3178, 3340, 4363, 4526, 7120, 9219, 9756, 11523, 11857, 13125, 13488, 14438, 15434)

$$c_{8190} = x_5 + x_1x_2x_3x_5 + x_1x_2x_3x_4x_5 + x_6 + x_1x_5x_8 + x_2x_3x_4x_6x_{12} + x_2x_4x_6x_7x_{11}x_{12} + x_3x_4x_9x_{11}x_{12} + x_1x_2x_4x_9x_{13} + x_2x_3x_4x_6x_8x_9x_{13} + x_5x_7x_8x_9x_{10}x_{12}x_{13} + x_1x_2x_{11}x_{14} + x_3x_4x_5x_{10}x_{11}x_{14} + x_1x_2x_9x_{11}x_{12}x_{14} + x_1x_5x_7x_{10}x_{11}x_{12}x_{14} + x_1x_3x_7x_9x_{10}x_{13}x_{14} + x_5x_6x_8x_{11}x_{13}x_{14} + x_2x_3x_6x_7x_{12}x_{13}x_{14} + x_2x_4x_7x_{11}x_{12}x_{13}x_{14}.$$

c_{4378}: (16 monomials; masks 876, 1359, 1369, 2049, 2226, 2315, 2651, 3788, 4825, 5984, 6406, 10304, 11020, 11544, 12595, 12686)

$$c_{4378} = x_3x_4x_6x_7x_9x_{10} + x_1x_2x_3x_4x_7x_9x_{11} + x_1x_4x_5x_7x_9x_{11} + x_1x_{12} + x_2x_5x_6x_8x_{12} + x_1x_2x_4x_9x_{12} + x_1x_2x_4x_5x_7x_{10}x_{12} + x_3x_4x_7x_8x_{10}x_{11}x_{12} + x_1x_4x_5x_7x_8x_{10}x_{13} + x_6x_7x_9x_{10}x_{11}x_{13} + x_2x_3x_9x_{12}x_{13} + x_7x_{12}x_{14} + x_3x_4x_9x_{10}x_{12}x_{14} + x_4x_5x_9x_{11}x_{12}x_{14} + x_1x_2x_5x_6x_9x_{13}x_{14} + x_2x_3x_4x_8x_9x_{13}x_{14}.$$

c_{4380}: (14 monomials; masks 366, 876, 1359, 1369, 2049, 2226, 2651, 4825, 6406, 10304, 11020, 11544, 12595, 12686)

$$c_{4380} = x_2x_3x_4x_6x_7x_9 + x_3x_4x_6x_7x_9x_{10} + x_1x_2x_3x_4x_7x_9x_{11} + x_1x_4x_5x_7x_9x_{11} + x_1x_{12} + x_2x_5x_6x_8x_{12} + x_1x_2x_4x_5x_7x_{10}x_{12} + x_1x_4x_5x_7x_8x_{10}x_{13} + x_2x_3x_9x_{12}x_{13} + x_7x_{12}x_{14} + x_3x_4x_9x_{10}x_{12}x_{14} + x_4x_5x_9x_{11}x_{12}x_{14} + x_1x_2x_5x_6x_9x_{13}x_{14} + x_2x_3x_4x_8x_9x_{13}x_{14}.$$

c_{4382}: (14 monomials; masks 876, 1359, 1369, 2049, 2226, 2651, 3697, 4825, 6406, 10304, 11020, 11544, 12595, 12686)

$$c_{4382} = x_3x_4x_6x_7x_9x_{10} + x_1x_2x_3x_4x_7x_9x_{11} + x_1x_4x_5x_7x_9x_{11} + x_1x_{12} + x_2x_5x_6x_8x_{12} + x_1x_2x_4x_5x_7x_{10}x_{12} + x_1x_5x_6x_7x_{10}x_{11}x_{12} + x_1x_4x_5x_7x_8x_{10}x_{13} + x_2x_3x_9x_{12}x_{13} + x_7x_{12}x_{14} + x_3x_4x_9x_{10}x_{12}x_{14} + x_4x_5x_9x_{11}x_{12}x_{14} + x_1x_2x_5x_6x_9x_{13}x_{14} + x_2x_3x_4x_8x_9x_{13}x_{14}.$$

c_{254}: (4 monomials; masks 4159, 8255, 10177, 12224)
$c_{254} = x_1x_2x_3x_4x_5x_6x_{13} + x_1x_2x_3x_4x_5x_6x_{14} + x_1x_7x_8x_9x_{10}x_{11}x_{14} + x_7x_8x_9x_{10}x_{11}x_{12}x_{14}$.

c_{314}: (4 monomials; masks 4159, 4191, 8255, 12192)
$c_{314} = x_1x_2x_3x_4x_5x_6x_{13} + x_1x_2x_3x_4x_5x_7x_{13} + x_1x_2x_3x_4x_5x_6x_{14} + x_6x_8x_9x_{10}x_{11}x_{12}x_{14}$.

c_{342}: (4 monomials; masks 4159, 4303, 8255, 12080)
$c_{342} = x_1x_2x_3x_4x_5x_6x_{13} + x_1x_2x_3x_4x_7x_8x_{13} + x_1x_2x_3x_4x_5x_6x_{14} + x_5x_6x_9x_{10}x_{11}x_{12}x_{14}$.

c_{350}: (4 monomials; masks 4159, 4551, 8255, 11832)
$c_{350} = x_1x_2x_3x_4x_5x_6x_{13} + x_1x_2x_3x_7x_8x_9x_{13} + x_1x_2x_3x_4x_5x_6x_{14} + x_4x_5x_6x_{10}x_{11}x_{12}x_{14}$.

c_{362}: (6 monomials; masks 4159, 4303, 4551, 8255, 12080, 12112)
$c_{362} = x_1x_2x_3x_4x_5x_6x_{13} + x_1x_2x_3x_4x_7x_8x_{13} + x_1x_2x_3x_7x_8x_9x_{13} + x_1x_2x_3x_4x_5x_6x_{14} + x_5x_6x_9x_{10}x_{11}x_{12}x_{14} + x_5x_7x_9x_{10}x_{11}x_{12}x_{14}$.

c_{4238}: (7 monomials; masks 4159, 4551, 4667, 8255, 8647, 11716, 12288)
$c_{4238} = x_1x_2x_3x_4x_5x_6x_{13} + x_1x_2x_3x_7x_8x_9x_{13} + x_1x_2x_4x_5x_6x_{10}x_{13} + x_1x_2x_3x_4x_5x_6x_{14} + x_1x_2x_3x_7x_8x_9x_{14} + x_3x_7x_8x_9x_{11}x_{12}x_{14} + x_{13}x_{14}$.

c_{4242}: (8 monomials; masks 4096, 4159, 4551, 5647, 8255, 8647, 12224, 12288)
$c_{4242} = x_{13} + x_1x_2x_3x_4x_5x_6x_{13} + x_1x_2x_3x_7x_8x_9x_{13} + x_1x_2x_3x_4x_{10}x_{11}x_{13} + x_1x_2x_3x_4x_5x_6x_{14} + x_1x_2x_3x_7x_8x_9x_{14} + x_7x_8x_9x_{10}x_{11}x_{12}x_{14} + x_{13}x_{14}$.

c_{4372}: (7 monomials; masks 4159, 5059, 7372, 8255, 9155, 10956, 12288)
$c_{4372} = x_1x_2x_3x_4x_5x_6x_{13} + x_1x_2x_7x_8x_9x_{10}x_{13} + x_3x_4x_7x_8x_{11}x_{12}x_{13} + x_1x_2x_3x_4x_5x_6x_{14} + x_1x_2x_7x_8x_9x_{10}x_{14} + x_3x_4x_7x_8x_{10}x_{12}x_{14} + x_{13}x_{14}$.

c_{4376}: (7 monomials; masks 4159, 5059, 7372, 8255, 9155, 11596, 12288)
$c_{4376} = x_1x_2x_3x_4x_5x_6x_{13} + x_1x_2x_7x_8x_9x_{10}x_{13} + x_3x_4x_7x_8x_{11}x_{12}x_{13} + x_1x_2x_3x_4x_5x_6x_{14} + x_1x_2x_7x_8x_9x_{10}x_{14} + x_3x_4x_7x_9x_{11}x_{12}x_{14} + x_{13}x_{14}$.

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