FIVE FALSE THRESHOLD TABLES IN COHEN'S CONJECTURES ON PRIMES AND CYCLIC NUMBERS

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Abstract. A positive integer n is *cyclic* if every group of order n is cyclic, equivalently (Szele) if $\gcd(n, \varphi(n)) = 1$. Cohen's *Conjectures about primes and cyclic numbers* collects sixty-three conjectures: cyclic-number analogues of classical conjectures about primes, together with new conjectures about the primes themselves. Many of them have a two-part shape: an asymptotic existence clause "for every k there is a threshold $N(k)$ such that …", followed by a printed table of specific values of $N(k)$. The existence clauses are out of reach; the tables are finite claims. We show that five of those tables are false: Conjectures 17, 20, 22, 28 and 29 fail at thirty-nine of their printed constants, and we exhibit an explicit witness for each. Four of the five refutations follow from counts that the source prints itself, in one case three lines below the conjecture they contradict, and in another from a sequence in the OEIS that predates the conjecture by nine years; we recompute all of them from the definitions rather than quoting them. The refutation of Conjecture 22 is horizon-free: because its threshold is *defined* to be least, one count at one index suffices, with no claim about large arguments. We also show that three further printed thresholds can be lowered unconditionally. What is not refuted, in every case, is the existence clause; we separate the two halves explicitly, because the recent literature reports Conjectures 17 and 20 as proved, and the only statement that literature prints for either conjecture is the existence clause alone. All statements are verified in Lean 4 by the kernel, with no compiled evaluation anywhere.

1. Introduction

A positive integer n is *cyclic* if every group of order n is cyclic. By a theorem of Szele this happens exactly when $\gcd(n, \varphi(n)) = 1$, and Cohen [1] takes that criterion as the definition. Every prime is cyclic, and the cyclic numbers $1, 2, 3, 5, 7, 11, 13, 15, 17, 19, 23, 29, 31, 33, 35, 37, \ldots$ (OEIS `A003277`) are a set of density zero that contains the primes and is otherwise built from the same local arithmetic. It is therefore natural to ask which conjectures about primes transfer, and [1] is a systematic pass through that question: sixty-three conjectures, some about cyclic numbers and some about primes — of the five treated here, only Conjecture 20 is about cyclic numbers — checked numerically against a sieve of the 28 488 167 cyclic numbers below 10^8 and the 50 847 534 primes below 10^9. (The conjectures are numbered 2–17, 19–20 and 22–66, the counter being shared with a theorem and two remarks, so the largest label is 66 but the count is 63.)

Many of the conjectures in [1] have the shape

(1) *for every k there exists $N(k)$ such that for all n past $N(k)$ the interval $I(n)$ contains at least k objects. In particular $N(2) = \ldots$, $N(3) = \ldots$, $N(4) = \ldots$*

The first half is an asymptotic statement. At $k = 1$ it is already, for the two conjectures of Section 4, Oppermann's conjecture in asymptotic form, so it is out of reach: no unconditional theorem produces a prime in every interval of length $x^{1/2}$ around x. The second half is different in kind. Each printed constant is a finite claim about a specific integer, refutable by exhibiting one index and counting. This paper is about the second half.

The status of that second half is not idle curiosity. Pomerance [5] settles several of Cohen's conjectures — among them the infinitude of twin cyclics, cyclic triples, and the Sophie Germain conjectures 36 and 37 — and writes that "somewhat more difficult are some of the short-interval conjectures" of [1] — that is, exactly the ones treated here. A counterexample to one further conjecture of [1] has been published by Ibarra [2], whose acknowledgment records the author of [1]

confirming that the search behind the disputed conjecture "resulted from an error in his computer code". So the tables of [1] are known to be, in at least one place, the output of a program with a bug in it, and they have not been audited.

What is settled here. Five of the printed tables are false, at thirty-nine constants in total.

- **Conjecture 17** (k-fold Oppermann for primes; Section 4). Ten of its twelve printed thresholds fail. The cleanest cell is $k = 2$: the conjecture asserts two primes in $[n^2, n^2+n]$ for every $n > 16$, and $[289, 306]$ contains exactly one prime, namely 293. That is eighteen integers, checkable by hand.
- **Conjecture 20** (the same for cyclic numbers; Section 4). Eleven of its twelve printed thresholds fail, under both readings of a printed "prime"/"cyclic" slip that is present in the version of record. The $k = 2$ cell contradicts a table that [1] prints three lines below the conjecture itself.
- **Conjecture 22** (k-fold Brocard; Section 5). Its threshold $B(k)$ is *defined*, three sentences earlier, to be the least index that works; fourteen of the seventeen printed values are not that. This refutation needs no horizon: if $v + 1$ is least, the property must fail at v, so a single count at a single index settles it.
- **Conjectures 28 and 29** (twin and cousin primes between consecutive cubes; Section 6). The value $N(10) = 11$ of Conjecture 28 fails at $n = 11$, where $(11^3, 12^3)$ contains nine twin-prime pairs and not ten; the three values $N(8) = N(9) = N(10) = 12$ of Conjecture 29 fail together at $n = 12$, where $(12^3, 13^3)$ contains seven cousin-prime pairs.

Four of these five refutations use numbers that [1] prints itself. The counterexample to Conjecture 28 is the eleventh entry of a list of twin-prime counts printed three sentences before the conjecture; that list is digit-identical to OEIS `A276119`, contributed in 2016 and extended to $n = 10\,000$ in an accompanying data file, so the refutation rests on an independent computation nine years older than the conjecture. The counterexample to Conjecture 20 is the fourth entry of a list of cyclic-number counts printed three lines below the conjecture. Section 3 recomputes all of these lists from the definitions rather than quoting them, because a refutation that leans on the source's own arithmetic is only worth something if that arithmetic is reproduced.

Three further printed thresholds — $N(11)$ and $N(12)$ of Conjecture 28 and $N(3)$ of Conjecture 29 — are shown to be non-least, by unconditional one-step implications (Proposition 6.5). These are *not* refutations: unlike $B(k)$ in Conjecture 22, the $N(k)$ of Conjectures 28 and 29 are not defined to be least, so a non-least value is not a false statement. They are recorded separately, and never counted among the thirty-nine, because the claim is strictly weaker.

The failures have a uniform shape, worth stating even though it is only an observation about the numbers.

Observation 1.1. In every one of the twenty-five failing cells of Conjectures 17, 20, 28 and 29, the counterexample is the *first* index that the printed threshold covers: $n = N(k)+1$ for Conjectures 17 and 20, which quantify over $n > N(k)$, and $n = N(k)$ for Conjectures 28 and 29, which quantify over $n \geq N(k)$. In twenty-three of those twenty-five cells the count at that index is exactly $k - 1$, one short of what is asserted; the two exceptions are $N(9)$ and $N(10)$ of Conjecture 29, which share a witness with $N(8)$. For Conjecture 22 the shape is the mirror image: the printed $B(k)$ exceeds a working index by exactly one in every one of the fourteen failing cells.

For Conjectures 17, 20, 28 and 29 all of this is consistent with a single off-by-one in the indexing of a table, and for Conjecture 22 a candidate index shift is visible on the page: the OEIS entry that [1] cites for its data has offset 0 and a conventional leading term, so its printed list of terms runs one place ahead of the index used in the conjecture. That shift would account for the printed $B(7) = B(8) = B(9) = 5$ and $B(10) = B(11) = 7$ and not for the other two groups, whose printed values exceed the least ones we find by two rather than by one (Section 8). We record the pattern, and make no claim about how the printed values were produced.

What is not settled here. Nothing in this paper touches an existence clause. Each conjecture treated below is the conjunction of an existence clause and a table; we refute the table and prove, formally, that the existence clause survives the refutation (Proposition 7.1). This matters beyond bookkeeping. Two recent papers report Conjectures 17 and 20 as proved, one of them — from its second version onward — marking Conjecture 17 as carrying a Lean certificate. Section 7 sets out what those papers state and what they do not: the only place either of them prints a Conjecture 17 or 20 statement has $N(k)$ bound by an existential quantifier, and Cohen's printed constants appear in neither paper. We therefore do not contradict them, and they do not bear on the tables. Section 8 collects everything else this paper leaves open, including the corrected thresholds, which involve an infinite tail and are therefore stated only as computations, outside the formal development.

Every claim below marked as a theorem or proposition has been verified in Lean 4 against *Mathlib*; Section 9 says exactly what that means, which claims rest on exhaustive enumeration, and how large each enumeration is.

2. Preliminaries

2.1. **Objects.** Throughout, p_m denotes the m-th prime in the indexing of [1], so $p_1 = 2$; and φ is Euler's function.

Definition 2.1. An integer $n \geq 1$ is *cyclic* if $\gcd(n, \varphi(n)) = 1$. We write $C(N)$ for the number of cyclic numbers in $[1, N]$.

For integers $a \leq b$ put

$$\pi[a,b] = \#\{p \in [a,b] : p \text{ prime}\}, \qquad \pi(a,b) = \#\{p \in (a,b) : p \text{ prime}\},$$

and let $\kappa[a,b]$, $\kappa(a,b)$ be the corresponding counts of cyclic numbers. For $a < b$ let

$$T(a,b) = \#\{p : a < p,\ p+2 < b,\ p \text{ and } p+2 \text{ prime}\}$$

be the number of twin-prime pairs *between* a and b; the convention that both members lie strictly inside is the one fixed by the source's own example, in which $(3,5)$ and $(5,7)$ are the two pairs between 1 and 8. Following [1], which follows Wolf, a *cousin* pair is a pair of *consecutive* primes at distance 4 and a *sexy* pair a pair of consecutive primes at distance 6; so $(3,7)$ is not a cousin pair, 5 lying between. Write $K(a,b)$ and $S(a,b)$ for the counts of cousin and sexy pairs between a and b, defined like $T(a,b)$ with the extra requirement that no prime lie strictly between the two members.

The consecutiveness requirement is not decorative and the twin case does not need it.

Proposition 2.2. *For all $a < b$, requiring the two members of a twin pair to be consecutive primes changes nothing: the two counts agree on every interval. For cousins the requirement is load-bearing: between 1^3 and 2^3 there is one pair of primes at distance 4, namely $(3,7)$, and no cousin pair.*

Proof. If p and $p+2$ are prime and $p+1$ were prime, then two of $p, p+1, p+2$ are even, forcing $p = 2$ and $p+2 = 4$, which is not prime. The cousin statement is the enumeration of $\{2, \ldots, 7\}$. □

The second half of Proposition 2.2 is exactly why the first entry of the cousin-count list printed in [1] is 0 and not 1 (Table 1 below), so the convention read off from the source is confirmed by the source's own data.

2.2. **The conjectures, as printed.** We use the following predicates, each transcribed from the sentence of [1] that contains it.

Definition 2.3. For $k, N, n \in \mathbb{N}$:

$$\begin{aligned}
\mathrm{O}_{17}(k,N): &\quad \forall\, n > N,\ \pi[n^2-n,\, n^2] \geq k \ \text{ and } \ \pi[n^2,\, n^2+n] \geq k;\\
\mathrm{O}_{20}(k,N): &\quad \forall\, n > N,\ \kappa[n^2-n,\, n^2] \geq k \ \text{ and } \ \kappa[n^2,\, n^2+n] \geq k;\\
\mathrm{O}'_{20}(k,N): &\quad \forall\, n > N,\ \kappa[n^2-n,\, n^2] \geq k \ \text{ and } \ \pi[n^2,\, n^2+n] \geq k;\\
\mathrm{B}_{22}(k,n): &\quad \forall\, m \geq n,\ \pi\big(p_m^2,\, p_{m+1}^2\big) \geq k;\\
\mathrm{Q}_{28}(k,N): &\quad \forall\, n \geq N,\ T\big(n^3, (n+1)^3\big) \geq k;\\
\mathrm{Q}_{29}(k,N): &\quad \forall\, n \geq N,\ K\big(n^3, (n+1)^3\big) \geq k.
\end{aligned}$$

Three conventions in Definition 2.3 are taken from the source and are worth justifying, since the refutations are sensitive to all three.

Strict versus non-strict. Conjectures 17 and 20 are printed with "for all $n > N(k)$", Conjectures 28 and 29 with "for all $n \geq N(k)$". The non-strict reading in the latter two is the source's own, not an interpretation imposed to create a counterexample:

Proposition 2.4. *$T(4^3, 5^3) = 3$ and $T(7^3, 8^3) = 4$. Consequently, read with "$n \geq N(k)$", the printed values $N(4) = 5$ and $N(5) = 8$ of Conjecture 28 cannot be lowered, there being only three twin pairs between 4^3 and 5^3 and only four between 7^3 and 8^3; whereas read with "$n > N(k)$" both could be lowered by one, the failing index falling outside the range they constrain.*

Open versus closed. Conjectures 17 and 20 print closed intervals $[n^2 - n, n^2]$ and $[n^2, n^2 + n]$; Conjectures 22, 28 and 29 say "between x and y" and we read them as open. For Conjecture 17 the distinction is empty, since n^2 and $n^2 \pm n = n(n \pm 1)$ are composite for $n > 1$. For Conjecture 20 it is not empty in general — composite cyclic numbers exist, 15 being the smallest — but it is empty at the cell that does the work: none of 20, 25, 30 is cyclic.

Cyclicity is a real condition. 15 is cyclic and composite, 4 and 9 are not cyclic, and 1 is; so κ is neither the prime-counting function nor everything. All four facts are part of the formal development.

2.3. **Refuting a least-threshold claim without a horizon.** Conjecture 22 is the one conjecture below whose threshold is defined to be least, and that makes its refutation cheaper rather than dearer.

Lemma 2.5. *Let P be any predicate on $\mathbb{N}$ and $v \in \mathbb{N}$. If $P(v)$ holds, then $v + 1$ is not the least n with the property that $P(m)$ holds for all $m \geq n$.*

Proof. If $P(m)$ holds for all $m \geq v + 1$ and also at v, then it holds for all $m \geq v$, so v belongs to the set of which $v + 1$ was supposed to be a lower bound; but $v < v + 1$. □

Lemma 2.5 makes no statement whatsoever about large m: it converts a claim about an infinite tail into one evaluation of P. Its hypothesis is what does the work, and it is not vacuous — 2 genuinely is the least n with $m \geq n \Rightarrow m \geq 2$, because the hypothesis fails at $v = 1$, whereas 3 is not, because it holds at $v = 2$. Both halves of that sanity check are part of the formal development.

3. The source's own numbers, recomputed

Four of the five refutations below rest on counts printed in [1] itself. That is a strength only if those counts are recomputed from the definitions rather than copied, so this section records the recomputation first, before any refutation is claimed. All lists here agree digit for digit with the lists printed in [1]; where an OEIS entry covers the same data, they agree with it too.

Proposition 3.1. *For $n = 1, \ldots, 13$, the numbers $T(n^3, (n+1)^3)$, $K(n^3, (n+1)^3)$ and $S(n^3, (n+1)^3)$ of twin, cousin and sexy prime pairs between consecutive cubes are the entries of Table 1.*

n	1	2	3	4	5	6	7	8	9	10	11	12	13
T	2	2	3	3	5	5	4	6	5	11	**9**	12	11
K	0	2	2	5	3	5	8	3	11	7	12	**7**	15
S	0	0	3	2	5	6	7	11	7	15	11	12	19

TABLE 1. Twin, cousin and sexy prime pairs between n^3 and $(n+1)^3$, recomputed from the definitions. All three rows agree with the lists printed in [1]; the twin row agrees with OEIS `A276119`. The two entries in bold are the counterexamples of Section 6.

The twin row of Table 1 is the sequence `A276119` of the OEIS, contributed in 2016, with an accompanying data file reaching $n = 10\,000$; its eleventh term is 9. The cousin row is, as far as we can determine, in no OEIS entry: a search on the term string returns nothing (28 August 2026), so the list printed in [1] and the recomputation here appear to be the only two independent computations of it on record. The sexy row is a control: it is the data behind Conjecture 30 of [1], and we refute no entry of that conjecture's table.

Proposition 3.2. *For $n = 2, \dots, 25$, the smaller of $\kappa(n^2 - n, n^2)$ and $\kappa(n^2, n^2 + n)$ is*

$$1,\ 1,\ 2,\ \mathbf{1},\ 2,\ 2,\ 2,\ 3,\ 3,\ 3,\ 3,\ 4,\ 3,\ 5,\ 5,\ 4,\ 6,\ 5,\ 6,\ 7,\ 6,\ 7,\ 9,\ 7.$$

This is the list printed in [1] three lines below Conjecture 20, and it is reproduced here for exactly one reason: its fourth entry, at $n = 5$, is 1, and Conjecture 20 asserts that every $n > 4$ has two. We note an editorial detail. The sentence carrying the list, on page 13 of the version of record, reads

> For $n = 2, 3, \dots, 25$, the lesser of the number of cyclics in $(n^2 - n, n^2)$ and the number of cyclics in $(n^2, n^2 + n)$ is 1, 1, 2, 1, 2, 2, 2, 3, 3, 3, 3, 4, 3, 5, 5, 4, 6, 5, 6, 7, 6, 7, 9, 7, and 7.

That is twenty-five entries for a stated range of twenty-four values of n. The first twenty-four are exactly the values displayed above, so the surplus entry is the last one, standing beyond the stated range; we do not examine it.

Proposition 3.3. *For $m = 1, \dots, 15$, the number $\pi(p_m^2, p_{m+1}^2)$ of primes strictly between consecutive prime squares is*

$$2,\ 5,\ 6,\ 15,\ 9,\ 22,\ 11,\ 27,\ 47,\ 16,\ 57,\ 44,\ 20,\ 46,\ 80.$$

The same statement holds with p_m replaced by the m-th entry of the list

$$2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ 31,\ 37,\ 41,\ 43,\ 47,\ 53,$$

which is proved to be the list of the first sixteen primes.

This is the sequence `A050216` of the OEIS, which [1] cites as the source of Conjecture 22's data and which in turn cites [1]. The second sentence of Proposition 3.3 is not padding: without it the statement would be about a hard-coded list of integers rather than about the primes, and the identification of the list with $(p_m)_{m \le 16}$ is what makes Section 5 a statement about Conjecture 22. The OEIS entry has offset 0, and its name records that its first term $a(0) = 2$ is conventional; for $m \ge 1$ its term $a(m)$ is exactly the count $\pi(p_m^2, p_{m+1}^2)$ displayed above, so its printed list of terms runs one place ahead of the index m used here.

3.1. **Anchors on the cyclic side.** Conjecture 20 is about cyclic numbers, and a computation about cyclic numbers has to be anchored: the sieve behind [1] is known to have had at least one error [2], and our own could have others. The following are places where the definition used here is pinned against numbers computed by other people.

Proposition 3.4. *The cyclic numbers below* 38 *are* $1, 2, 3, 5, 7, 11, 13, 15, 17, 19, 23, 29, 31, 33, 35, 37$*;* $C(10) = 5$*,* $C(100) = 37$ *and* $C(500) = 166$*; and for* $n = 1, \dots, 10$ *the number of cyclic numbers strictly between* c_n^2 *and* c_{n+1}^2*, where* c_n *is the* n*-th cyclic number, is*

$$2,\ 2,\ 6,\ 8,\ 25,\ 16,\ 17,\ 21,\ 22,\ 56.$$

The first list is both the opening list of [1] and the head of OEIS `A003277`; the last is a table printed in [1]. The strongest anchor available is a different one. Cohen's Conjecture 66 asserts that $C_\sigma(m+n) \le C_\sigma(m) + C_\sigma(n)$ for all $1 \le m \le n$, where $C_\sigma(N)$ counts the *Sophie Germain cyclic* numbers $c \le N$, those for which $2c+1$ is again cyclic. Ibarra [2] refuted it at $m = 31$, $n = 3928$, with a Lean proof distributed alongside the paper. We re-derive that refutation from the definitions used here.

Proposition 3.5. $C_\sigma(31) = 10$*, exactly eleven Sophie Germain cyclic numbers lie in* $(3928, 3959]$*, and therefore*

$$C_\sigma(3959) = C_\sigma(3928) + 11 > C_\sigma(31) + C_\sigma(3928),$$

so Conjecture 66 of [1] *is false.*

The result is Ibarra's and the credit is his; the point of repeating it is that our independently written cyclicity predicate, and an independently written splitting identity for C_σ, reproduce someone else's machine-checked counterexample. Like [2], the argument never evaluates $C_\sigma(3928)$: it uses the value at 31 and the size of a thirty-one element window, which is what keeps the computation small.

3.2. **How the counts are computed, and why the statements do not depend on it.** Every count in this paper is *stated* with the standard primality predicate of *Mathlib* (or with φ), and *computed* with trial division against every integer in $[2, 199]$, the two being joined by a proved equivalence valid below $200^2 = 40\,000$. Every interval occurring below has all its endpoints under that bound. The reason for the detour is speed: the library's primality decision procedure does not reduce inside the kernel at this scale, one interval of 468 integers failing to finish in two minutes, where trial division does the same interval in seconds. The bound is load-bearing rather than decorative, and the formal development records why: trial division against $[2, 199]$ declares $44\,521 = 211^2$ prime. Because the equivalence carries the hypothesis $n < 40\,000$, no statement in this paper depends on the trial-division code being the intended notion of primality.

4. Conjectures 17 and 20: thresholds between consecutive squares

Oppermann's conjecture asserts that for $n > 1$ each of the intervals $(n^2 - n, n^2)$ and $(n^2, n^2 + n)$ contains a prime. Conjecture 17 of [1] is its k-fold form, printed as follows.

Conjecture 4.1 ([1, Conjecture 17])**.** For every $k \in \mathbb{N}$, there exists $N(k) \in \mathbb{N}$ such that for all $n > N(k)$, there exist at least k primes in $[n^2 - n, n^2]$ and another at least k primes in $[n^2, n^2 + n]$. In particular, $N(2) = 16$, $N(3) = 36$, $N(4) = 46$, $N(5) = 76$, $N(6) = N(7) = 79$, $N(8) = 85$, $N(9) = 118$, $N(10) = 136$, $N(11) = N(12) = 155$, $N(13) = 188$.

Theorem 4.2. *For each of the ten rows of Table 2, the stated interval contains exactly the stated number of primes; consequently* $\mathrm{O}_{17}(k, N(k))$ *fails for* $k = 2, 3, 4, 5, 7, 8, 9, 10, 12, 13$*. In particular Conjecture 4.1 is false as printed.*

Proof sketch. Each row is one exhaustive enumeration of a closed interval of $n+1$ integers, the largest being the 190 integers of $[35532, 35721]$; the ten intervals together contain 955 integers. Each integer is tested for primality by trial division against $[2, 199]$ and the resulting count is transported to the count of actual primes by the equivalence of Section 3, whose hypothesis holds since $35\,721 < 40\,000$. Given the count, the refutation is immediate: $\mathrm{O}_{17}(k, N(k))$ applied at $n = N(k)+1$ asserts $k \le c$ for the tabulated $c = k - 1$. The final sentence is the projection of the printed conjecture, which is the conjunction of its existence clause with its twelve tabulated assertions, onto the second conjunct. □

k	$N(k)$	n	interval	#primes
2	16	17	$[289, 306]$	1
3	36	37	$[1332, 1369]$	2
4	46	47	$[2162, 2209]$	3
5	76	77	$[5929, 6006]$	4
7	79	80	$[6400, 6480]$	6
8	85	86	$[7310, 7396]$	7
9	118	119	$[14161, 14280]$	8
10	136	137	$[18769, 18906]$	9
12	155	156	$[24180, 24336]$	11
13	188	189	$[35532, 35721]$	12

TABLE 2. The ten failing cells of Conjecture 4.1. In each row $n = N(k) + 1$ is the first index the printed threshold covers, the interval is one of the two half-intervals at that n, and the count falls exactly one short of k.

Two things should be said about the negative result before it is believed. The first is that it is one-sided: at $n = 17$ the *other* half-interval does contain two primes, so the conjecture is not being failed by a modelling slip that would make it fail everywhere. The second is that the headline count is tight in both directions.

Proposition 4.3. *$\pi[289, 306] \neq 0$, and 293 is a prime in $[289, 306]$; $\pi[289, 306] < 2$, which is precisely the failure of Conjecture 4.1 at $k = 2$; and $\pi[272, 289] \geq 2$, the other half-interval at $n = 17$.*

The two printed values of Conjecture 4.1 that are not refuted are $N(6) = 79$ and $N(11) = 155$; see Section 8 for what we do and do not say about them.

Conjecture 20 transfers the same statement to cyclic numbers.

Conjecture 4.4 ([1, Conjecture 20])**.** For every $k \in \mathbb{N}$, there exists $N(k) \in \mathbb{N}$ such that for all $n > N(k)$, there exist at least k cyclic numbers in the interval $[n^2 - n, n^2]$ and another at least k prime numbers in the interval $[n^2, n^2+n]$. In particular, $N(2) = 4$, $N(3) = 7$, $N(4) = 13$, $N(5) = 16$, $N(6) = 18$, $N(7) = 21$, $N(8) = 25$, $N(9) = 31$, $N(10) = N(11) = 32$, $N(12) = 40$, $N(13) = 44$.

The word "prime" in the second interval is a slip for "cyclic". The conjecture sits in a run of cyclic-number analogues, and the list [1] prints three lines below it tabulates the smaller of the two counts of *cyclic* numbers in the two intervals (Proposition 3.2), so the intended reading is the cyclic one. The slip is nevertheless present in the version of record, so we formalise both readings, O_{20} (both intervals cyclic) and O'_{20} (literally as printed), and refute the disputed cell under both.

Theorem 4.5. *For each of the eleven rows of Table 3, the stated interval contains exactly the stated number of cyclic numbers; consequently $\mathrm{O}_{20}(k, N(k))$ fails for $k = 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13$, and Conjecture 4.4 is false as printed. Moreover $\pi[25, 30] = 1$, so $\mathrm{O}'_{20}(2, 4)$ fails as well: the printed slip does not save the conjecture.*

Proof sketch. Each row is an exhaustive enumeration of a closed interval of $n + 1$ integers — 279 integers over all twelve enumerations, including the prime count in $[25, 30]$ — in which each integer is tested by evaluating $\gcd(n, \varphi(n))$ directly. No primality bridge is needed for the cyclic counts; the one prime count is handled as in Theorem 4.2. □

What makes the $k = 2$ cell of Theorem 4.5 more than an isolated computation is that the source contradicts itself there. Proposition 3.2 recomputes the list that [1] prints three lines below Conjecture 4.4: for $n = 2, \ldots, 25$, the smaller of the two cyclic counts. Its entry at $n = 5$ is 1.

k	$N(k)$	n	interval	#cyclic
2	4	5	$[20, 25]$	1
3	7	8	$[56, 64]$	2
4	13	14	$[196, 210]$	3
5	16	17	$[272, 289]$	4
6	18	19	$[342, 361]$	5
7	21	22	$[462, 484]$	6
8	25	26	$[650, 676]$	7
9	31	32	$[1024, 1056]$	8
11	32	33	$[1056, 1089]$	10
12	40	41	$[1640, 1681]$	11
13	44	45	$[1980, 2025]$	12

TABLE 3. The eleven failing cells of Conjecture 4.4. Again $n = N(k) + 1$ in every row and the count is exactly $k - 1$. Only $N(10) = 32$ is not refuted.

Conjecture 4.4 asserts that both counts are at least 2 for every $n > 4$. The two statements cannot both hold, and the recomputation says which one is right: the interval $[20, 25]$ contains the single cyclic number 23, and $[25, 30]$ contains the single cyclic number 29, which is also its single prime. The open and closed readings agree here because 20, 25 and 30 are all non-cyclic.

5. Conjecture 22: thresholds that are not least

Brocard's conjecture asserts that at least four primes lie between p_m^2 and p_{m+1}^2 for $m \geq 2$. Cohen's k-fold form is preceded, in the paragraph immediately before it, by a definition, and the definition is what the conjecture is about:

> For $k \in \mathbb{N}$, $k \geq 4$, define $B(k)$ to be the smallest $n \in \mathbb{N}$ such that, for all $m \in \mathbb{N}$ with $m \geq n$, there are always at least k primes between p_m^2 and p_{m+1}^2.

The word *smallest* is the source's own; no emphasis has been added.

Conjecture 5.1 ([1, Conjecture 22])**.** Let $B(4) = 2$, $B(5) = 2$, $B(6) = 3$, $B(7) = B(8) = B(9) = 5$, $B(10) = B(11) = 7$, $B(12) = B(13) = B(14) = B(15) = B(16) = 10$, $B(17) = B(18) = B(19) = B(20) = 13$. Then for $k = 4, \ldots, 20$ and for every $n \in \mathbb{N}$ such that $n \geq B(k)$, there exist at least k primes between p_n^2 and p_{n+1}^2. More generally, for every $k \in \mathbb{N}$, there exists $B(k) \in \mathbb{N}$ such that for every $n \in \mathbb{N}$ with $n \geq B(k)$, there exist at least k primes between n^2 and $(n + 1)^2$.

The first two sentences of the conjecture therefore make two assertions: that the listed numbers are the values of B, and that they are working thresholds. We refute the first only. Nothing here bears on the second: a working threshold stays a working threshold when it is enlarged, so showing that a printed value is not least says nothing about whether it works. The two readings are kept apart formally, and only the first is refuted. Nothing here bears on the third sentence either — an existence clause of Legendre's kind, about intervals $(n^2, (n + 1)^2)$ rather than about intervals between prime squares — which is untouched throughout.

Theorem 5.2. *For $7 \leq k \leq 20$ the printed value $B(k)$ of Conjecture 5.1 is not the least n such that at least k primes lie between p_m^2 and p_{m+1}^2 for all $m \geq n$. More precisely, and unconditionally, for each such k*

$$\mathrm{B}_{22}\big(k, B(k)\big) \implies \mathrm{B}_{22}\big(k, B(k) - 1\big),$$

so whatever happens for large m, the printed threshold can be lowered by one step for free. In particular Conjecture 5.1 is false as printed, at fourteen of its seventeen constants.

Proof sketch. Four counts do all the work, and they are the entries of Proposition 3.3 at $m = 4, 6, 9, 12$:

$$\pi(7^2, 11^2) = 15, \quad \pi(13^2, 17^2) = 22, \quad \pi(23^2, 29^2) = 47, \quad \pi(37^2, 41^2) = 44.$$

Each is an exhaustive enumeration of an open interval — 71, 119, 311 and 311 integers — with primality by trial division below 40 000 as in Section 3, and each identification $p_4 = 7, \dots, p_{13} = 41$ is proved from the library's characterisation of the m-th prime rather than assumed. For $7 \le k \le 9$ we have $B(k) = 5$ and $15 \ge k$ at $m = 4 = B(k) - 1$; for $k = 10, 11$, $B(k) = 7$ and $22 \ge k$ at $m = 6$; for $12 \le k \le 16$, $B(k) = 10$ and $47 \ge k$ at $m = 9$; for $17 \le k \le 20$, $B(k) = 13$ and $44 \ge k$ at $m = 12$. The displayed implication is then immediate by cases on $m = B(k) - 1$ or $m \ge B(k)$, and the failure of leastness is Lemma 2.5 applied with $v = B(k) - 1$. □

We stress what this proof does not use. It contains no claim about $\pi(p_m^2, p_{m+1}^2)$ for large m — indeed no claim for any m other than the four listed — and it does not require Conjecture 5.1 to be true anywhere. A least-threshold assertion is refuted by one evaluation below the threshold; that is Lemma 2.5, and it is the reason this conjecture, superficially the most infinitary of the five, is the cheapest to settle. The three printed values $B(4) = 2$, $B(5) = 2$ and $B(6) = 3$ are not refuted.

6. Conjectures 28 and 29: twin and cousin primes between cubes

Conjecture 6.1 ([1, Conjecture 28])**.** For every $n \in \mathbb{N}$, the number of pairs of twin primes between n^3 and $(n+1)^3$ is never less than two. More generally, for every $k \in \mathbb{N}$, there exists $N(k) \in \mathbb{N}$ such that for all $n \ge N(k)$ there are at least k pairs of twin primes between n^3 and $(n+1)^3$. Specifically, $N(1) = N(2) = 1$, $N(3) = 3$, $N(4) = 5$, $N(5) = 8$, $N(6) = N(7) = N(8) = N(9) = 10$, $N(10) = 11$, $N(11) = 13$, $N(12) = \cdots = N(16) = 15$, and $N(17) = 20$.

Theorem 6.2. *$T(11^3, 12^3) = 9$. Since $11 \ge 11$, the printed value $N(10) = 11$ of Conjecture 6.1 is false: $\mathrm{Q}_{28}(10, 11)$ fails. In particular Conjecture 6.1 is false as printed.*

Proof sketch. One exhaustive enumeration of the 396 integers strictly between 1331 and 1728, with primality by trial division below 40 000. The nine pairs are

$$(1427, 1429),\ (1451, 1453),\ (1481, 1483),\ (1487, 1489),\ (1607, 1609),$$
$$(1619, 1621),\ (1667, 1669),\ (1697, 1699),\ (1721, 1723),$$

and nine is not at least ten. □

The interest of Theorem 6.2 is that the refutation is printed in [1] three sentences before the conjecture: the eleventh entry of its own twin-prime list is 9 (Table 1). That list is digit-identical to OEIS `A276119`, whose eleventh term has been on record since 2016, so the refutation does not depend on our arithmetic at all — only on reading the conjecture against data the source and the OEIS agree on. The correct value is discussed in Section 8.

Conjecture 6.3 ([1, Conjecture 29])**.** For $n \in \mathbb{N}$ with $n > 1$, the number of pairs of cousin primes between n^3 and $(n+1)^3$ is never less than two. More generally, for every $k \in \mathbb{N}$, there exists $N(k) \in \mathbb{N}$ such that, for all $n \ge N(k)$, there are at least k pairs of cousin primes between n^3 and $(n+1)^3$. Specifically, $N(1) = N(2) = 2$, $N(3) = 8$, $N(4) = N(5) = N(6) = N(7) = 9$, $N(8) = N(9) = N(10) = 12$.

Theorem 6.4. *$K(12^3, 13^3) = 7$. Since $12 \ge 12$, the three printed values $N(8) = N(9) = N(10) = 12$ of Conjecture 6.3 fail together: $\mathrm{Q}_{29}(k, 12)$ is false for $k = 8, 9, 10$. In particular Conjecture 6.3 is false as printed.*

Proof sketch. One exhaustive enumeration of the 468 integers strictly between 1728 and 2197. For each candidate p the test is that p and $p + 4$ are prime and that neither $p + 1$, $p + 2$ nor $p + 3$ is — the consecutiveness condition of Section 2 — so the seven pairs are

$$(1783, 1787),\ (1867, 1871),\ (1873, 1877),\ (1993, 1997),\ (1999, 2003),\ (2083, 2087),\ (2137, 2141).$$

Seven is not at least eight, nine or ten. □

Here too the refuting count is the twelfth entry of a list printed in [1] itself (Table 1), though in this case with no OEIS entry to corroborate it. The consecutiveness condition is not a device of ours: dropping it changes the first entry of that same printed list from 0 to 1 (Proposition 2.2), so the source's data fixes the convention.

Finally, the same slip shows through in three printed values that survive it. These are not refutations — $N(k)$ in Conjectures 6.1 and 6.3 is not defined to be least, so a non-least printed value is not a false statement — and we record them separately for that reason.

Proposition 6.5. *Unconditionally,*

$$\mathrm{Q}_{28}(11,13) \Rightarrow \mathrm{Q}_{28}(11,12), \qquad \mathrm{Q}_{28}(12,15) \Rightarrow \mathrm{Q}_{28}(12,14), \qquad \mathrm{Q}_{29}(3,8) \Rightarrow \mathrm{Q}_{29}(3,4).$$

In particular, whatever happens for large n, the printed thresholds $N(11) = 13$ and $N(12) = 15$ of Conjecture 6.1 and $N(3) = 8$ of Conjecture 6.3 are not the smallest that work.

Proof sketch. The counts behind the three implications are $T(12^3, 13^3) = 12$, $T(14^3, 15^3) = 12$, and $K(n^3, (n+1)^3) \geq 3$ for each of $n = 4, 5, 6, 7$, with exact values $5, 3, 5, 8$; the enumerations are of 468, 630, and $60 + 90 + 126 + 168$ integers respectively. Each implication then follows by splitting the hypothesis $n \geq N - 1$ (resp. $n \geq 4$) into the finitely many small cases and the tail, on which the hypothesis applies verbatim. □

7. What the recent literature certifies

Two recent papers, sharing a first author, report Conjectures 4.1 and 4.4 as proved. Since Section 4 shows both false as printed, the discrepancy calls for an explanation, and it is a simple one: the only statement either paper prints for these two conjectures is the existence clause of (1), with the threshold bound by an existential quantifier. We set out the evidence, because the point is easy to state and easy to get wrong in either direction.

(i) The classification. Table 3 of [3] classifies conjectures of [1] by status. Its rows are headed `conjecture`, |itn|, "O or C", "P or R", "correct?" and "certified?", and the paragraph below the table says what they mean: the number of iterations of the authors' loop, then "whether the conjecture was open or already closed (O/C), whether the conjecture was settled by proof or refutation (P/R)", then a human judgement of correctness, then certification. In the column for Conjecture 17 the entries are O, P, "correct? Y", "certified? Y"; in the column for Conjecture 20 they are O, P, "correct? Y" and a blank certification cell. Those entries are present in version 2 (22 October 2025) and in version 3 (6 November 2025), the latest at the time of writing. They are *not* in version 1 (11 October 2025), whose Table 3 has only the three rows `conjecture`, |itn| and "P or R"; there Conjecture 17 and Conjecture 20 carry the single entry P. Version 3 also revises the surrounding claims: the summary of results went from twenty-two conjectures with five refutations (of Conjectures 35, 50, 51, 53 and 59) to twenty with three (35, 53 and 59).

(ii) Neither statement, nor the certificate, is printed. No version of [3] prints a statement of Conjecture 17 or 20, nor any Lean code; the theorems printed in full there concern other conjectures. We looked for the certificate as an artifact and did not find one: the electronic source of all three versions contains four files (a metadata file, a bibliography, the source and a style file) and no ancillary directory, whereas the Lean file accompanying [2] is distributed exactly that way; the five URLs occurring in the source and its bibliography are not code repositories; and the seven public repositories of the second author contain no Lean sources, the most recent commit to any of them predating the paper by more than two years. This is a negative search result and we record it as such, not as a claim that no artifact exists. These searches were run on 28 August 2026.

(iii) What "certified" is defined to mean there. The paragraph of [3] that explains Table 3, in versions 2 and 3, defines the certification column as recording whether the Lean certificate "is correct and conformant with the premises and conclusions of the corresponding proofs" — a human

judgement of conformance between a certificate and a proof as written, not a machine-checked link between a certificate and the sentence printed in [1]. Version 3 is also explicit that the formalisation was partial:

> In order to reduce human interaction, we had initially instructed OpenAI's Codex interface to reduce to "axiom" any proof step that it could find by means of a web search. This reduced the extent of `lean` checking (the formalization was partial) but made it possible to run a limited certification step at the end of our protocol

with an attached footnote reading "We have since enlisted the help of human experts in analytic number theory to help us verify the solutions of conjectures in […]. Some of them were found to be incorrect despite the limited formalization effort; and not all of them have been yet vetted." Version 3 carries on its first page the authors' own caution that "none of the 'proofs' mentioned herein should be taken as correct".

(iv) The one place a statement is printed, and what it says. The companion paper [4] — withdrawn on 6 November 2025, its author's comment reading "Proofs in this paper were AI-generated and I just found out some of them were incorrect. Therefore, I would like to withdraw it" — does print the theorems, and is the only place where either author prints a statement of Conjecture 17 or 20. For Conjecture 17 it reads: *for every $k \in \mathbb{N}$ there exists $N(k)$ such that for all $n > N(k)$, both intervals $[n^2 - n, n^2]$ and $[n^2, n^2 + n]$ contain at least k primes*, and the theorem for Conjecture 20 has the same shape with cyclic numbers. There $N(k)$ is bound by an existential quantifier; the constants $16, 36, 46, \ldots$ and $4, 7, 13, \ldots$ printed in [1] do not occur in the statement, in the proof, or anywhere else in that paper.

The conclusion is a matter of logic and not of judgement. The printed conjecture is a conjunction, and what those two papers report bears on one conjunct of it.

Proposition 7.1. *Write W_{17} for the existence clause "for every k there is N with $\mathrm{O}_{17}(k, N)$", and likewise $\mathrm{W}_{20}, \mathrm{W}_{28}, \mathrm{W}_{29}$; and write W_{22} for the assertion that the seventeen printed constants of Conjecture 5.1 are working thresholds. Then each printed conjecture implies its clause — Conjecture 4.1 $\Rightarrow \mathrm{W}_{17}$, and so on for 20, 22, 28, 29 — while each printed conjecture is false. So each printed conjecture fails on content that its clause does not contain, and every clause is left open.*

So the theorem printed in [4] and the theorems of this paper are about disjoint halves of the same sentences, and neither contradicts the other.

One link in the account above is an inference and should be labelled as one. That the Conjecture 17 column of Table 3 of [3] refers to the theorem printed in [4] is not stated anywhere: no version of [3] cites [4], and we did not ask the authors. It is inferred from the shared first author, from the shared conjecture numbering, and from [4] being the only place where either author prints a statement of Conjecture 17 or 20. Nothing in this paper depends on it. What is independent of the inference is this: no version of [3] prints a statement of Conjecture 17 or 20 or reproduces either table of constants — none of 118, 136, 155, 188 occurs anywhere in its source — so whatever theorem its entry records, that entry is nowhere on record as an assertion about the printed table. What does not survive is the reading under which Conjecture 17 or 20 of [1] has been settled as printed. We intend no criticism of the certification effort in [3], whose authors document their protocol's limits carefully and, in the current version, disclaim the proofs themselves; the distinction we draw is exactly the one Proposition 7.1 makes formal, and it is a distinction the two-part shape (1) invites anyone to miss.

Remark 7.2 (an observation about a printed argument, not a result of this paper). The proof of the Conjecture 17 theorem in [4] opens: "Fix $\theta = 23/42 > 1/2$. By the Iwaniec–Pintz theorem …, there exists $X_\theta \geq 2$ such that for all $x \geq X_\theta$ and all $y \leq x^\theta$, $\pi(x) - \pi(x - y) > y/(100 \log x)$." The quantifier on y is inverted. Short-interval theorems of this type bound $\pi(x) - \pi(x - y)$ from below for y *at least* a fixed power of x, not for all y below it, and as printed the displayed inequality is false: take $y = 2$ and x in the interior of a long prime gap, where the left side is 0 and the right

side is positive. The inequality is the sole engine of the proof. We add, for orientation, that the existence clause at $k = 1$ is Oppermann's conjecture in asymptotic form, and that applying a correct short-interval theorem at $y = n = x^{1/2}$ with $x = n^2$ is outside the range of every unconditional result known to us: the smallest exponent θ for which primes are known in the intervals $[x - x^\theta, x]$ for all large x is 0.525 [6], and $1/2 < 0.525$. (The passage quoted above cites [6] in its next sentence.) This paragraph is an argument about the literature, checkable by any reader; it is deliberately not part of the formal development, and it is not evidence that the existence clause is false.

8. What is not claimed

This paper refutes tables of constants. It is worth being explicit about everything it does not do, since the distinction between the two halves of (1) is the whole subject of Section 7.

The existence clauses are untouched. Proposition 7.1 states this formally. We have neither proved nor refuted any of W_{17}, W_{20}, W_{28}, W_{29}; at $k = 1$ the first is Oppermann's conjecture in asymptotic form.

Conjecture 5.1's working-threshold clause is untouched. Only the assertion that the printed values are the *least* working thresholds is refuted; the "Then …" clause, which asserts merely that they work, is implied by the corresponding statement for the true least thresholds and is left open.

The first sentences of Conjectures 6.1 and 6.3 are untouched. That every interval $(n^3, (n+1)^3)$ contains at least two twin pairs, and at least two cousin pairs for $n > 1$, is an infinite assertion; Table 1 is consistent with both over $n \le 13$ and we prove nothing beyond that.

The corrected thresholds are not proved, and cannot be by these methods. A corrected value of $N(k)$ or $B(k)$ is a statement about an infinite tail, and no finite computation establishes one. What our computations outside the formal development do establish is a range, and we state each range numerically. Over $n \le 169$ — the largest n with $(n+1)^3$ inside the sieve used, $170^3 = 4\,913\,000$ — the least working threshold is $N(10) = 12$ for Conjecture 6.1 and $N(8) = N(9) = N(10) = 13$ for Conjecture 6.3; apart from these and the three non-least entries of Proposition 6.5, every printed value of those two conjectures is least over that range. Over $m \le 60$, that is up to $p_{61}^2 = 283^2$, the printed $B(k)$ of Conjecture 5.1 are replaced by

$$B(7) = B(8) = B(9) = 4, \quad B(10) = B(11) = 6, \quad B(12) = \cdots = B(16) = 8,$$
$$B(17) = B(18) = B(19) = B(20) = 11,$$

one value for each k in each group of equal printed constants, not merely for the smallest k of the group. For Conjecture 4.1 the range is $n \le 2000$ and for Conjecture 4.4 it is $n \le 399$; in every refuted cell of those two the least working threshold over the range is the witness itself, the n tabulated in Table 2 or Table 3. All of these values are the output of a computation over a bounded range, are not part of the formal development, and carry no claim about large arguments.

Two printed values of Conjecture 4.1, one of Conjecture 4.4 and three of Conjecture 5.1 are not refuted. They are $N(6) = 79$ and $N(11) = 155$; $N(10) = 32$; and $B(4) = 2$, $B(5) = 2$, $B(6) = 3$. We do not assert that they are correct: that would again be a statement about an infinite tail. We assert only that our computations found no counterexample to them. For the two values of Conjecture 4.1 the computations outside the formal development locate a smaller working value over the range $n \le 2000$ (77 and 137), so those two would be non-least even if true; that observation is of the same kind as Proposition 6.5 but is not formalised here.

Nothing is claimed about the other conjectures of [1]. The data behind the paper's remaining finite-checkable conjectures was recomputed in a separate pass, outside the formal development, against an independently written sieve of the cyclic numbers below 10^9; that sieve reproduces the count $C(10^8) = 28\,488\,167$ on which [1] is built. We report one thing that pass turned up, and take it no further. The table of Conjecture 32 of [1] is printed as "$N(1) = N(2) = 1$, $N(3) = N(4) = 2$, $N(4) = N(5) = N(6) = N(7) = 3$, $N(8) = N(9) = N(10) = N(11) = N(12) = N(13) = 4$", in which the label $N(4)$ carries two different values. This is visible on the page of the version of record and needs no computation; it is a typographical matter, not of a kind with the five tables treated

above, and we do not say which of the two entries was intended. Beyond it we assert nothing about any other conjecture of [1], in either direction.

9. Verification

Every statement of this paper labelled a theorem or a proposition has been formally verified in Lean 4 (toolchain `v4.32.0`) against *Mathlib*, and the development contains no `sorry`; the repository reference is to be added at submission. All of it is checked by the Lean kernel: there is no use of compiled evaluation — no `native_decide` — at any point, so no claim here depends on trusting a compiler or a runtime, only the kernel and the three standard axioms `propext`, `Classical.choice` and `Quot.sound` (Lemma 2.5 and two of the controls use only two of them). What rests on exhaustive computation is precisely the interval counts, and they are small: the ten enumerations of Theorem 4.2 cover 955 integers, the twelve of Theorem 4.5 cover 279, the four of Theorem 5.2 cover 812, Theorems 6.2 and 6.4 cover 396 and 468, Proposition 6.5 covers 1 542, each row of Table 1 covers the 2 730 integers below 14^3, Proposition 3.2 covers 600, Proposition 3.3 covers 2 790, and the cyclic anchors of Propositions 3.4 and 3.5 cover at most 961 integers each, none of them evaluating a cumulative count beyond 500. Counts of primes are stated with *Mathlib*'s primality predicate and computed by trial division, the two joined by a proved equivalence below 40 000 (Section 3); counts of cyclic numbers are stated and computed with *Mathlib*'s φ. Every number the paper attributes to [1] or to an OEIS entry was independently recomputed by a separate program, written against those printed lists as assertions so that a disagreement would be a failure rather than a silent difference, and the two computations agree. The corrected thresholds and search ranges quoted in Section 8 come from that program alone and are not part of the formal development.

References

[1] J. E. Cohen, *Conjectures about primes and cyclic numbers*, Journal of Integer Sequences **28** (2025), Article 25.4.7, published 7 August 2025; also arXiv:2508.08335v1 (10 August 2025), the only version. Page numbers and quotations above are from the journal version, at `https://cs.uwaterloo.ca/journals/JIS/VOL28/Cohen/cohen41.pdf`; every number this paper disputes is identical in the two.

[2] J. A. Ibarra, *A counterexample to a subadditivity conjecture of Cohen for Sophie Germain cyclic numbers*, arXiv:2607.09793 (9 July 2026), with ancillary Lean 4 file `anc/cohen66.lean`.

[3] H. Le Duc and L. Liberti, *Mathematics with large language models as provers and verifiers*, arXiv:2510.12829; v1 11 October 2025, v2 22 October 2025, v3 6 November 2025. References above are to version 3 unless a version is named.

[4] H. Le Duc, *Conjectures about cyclic numbers: resolutions and counterexamples*, arXiv:2509.26138; withdrawn at v3 on 6 November 2025. Quotations above are from version 2 (1 October 2025). The name is as printed on the paper; the arXiv metadata for this item lists the author as "Duc Hieu Le".

[5] C. Pomerance, *Patterns for cyclic numbers*, to appear in *Integers* (Proceedings of the 2025 Integers Conference); preprint at `https://math.dartmouth.edu/~carlp/cyclic062225.pdf`.

[6] R. C. Baker, G. Harman and J. Pintz, *The difference between consecutive primes, II*, Proc. London Math. Soc. (3) **83** (2001), 532–562.

[7] OEIS Foundation Inc., *The On-Line Encyclopedia of Integer Sequences*, `https://oeis.org`; entries `A003277` (cyclic numbers), `A050216` (primes between consecutive prime squares) and `A276119` (twin prime pairs between consecutive cubes).